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Dynamics Days Europe

Chimera-like states in two interacting populations of heterogeneous quadratic integrate-and-fire neurons

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Overview

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- Single neuron
- Single population of neurons
  - Macroscopic field equations
  - Bifurcations
- Two interacting identical populations
  - Symmetric solutions
  - Non-symmetric solutions
- Conclusions

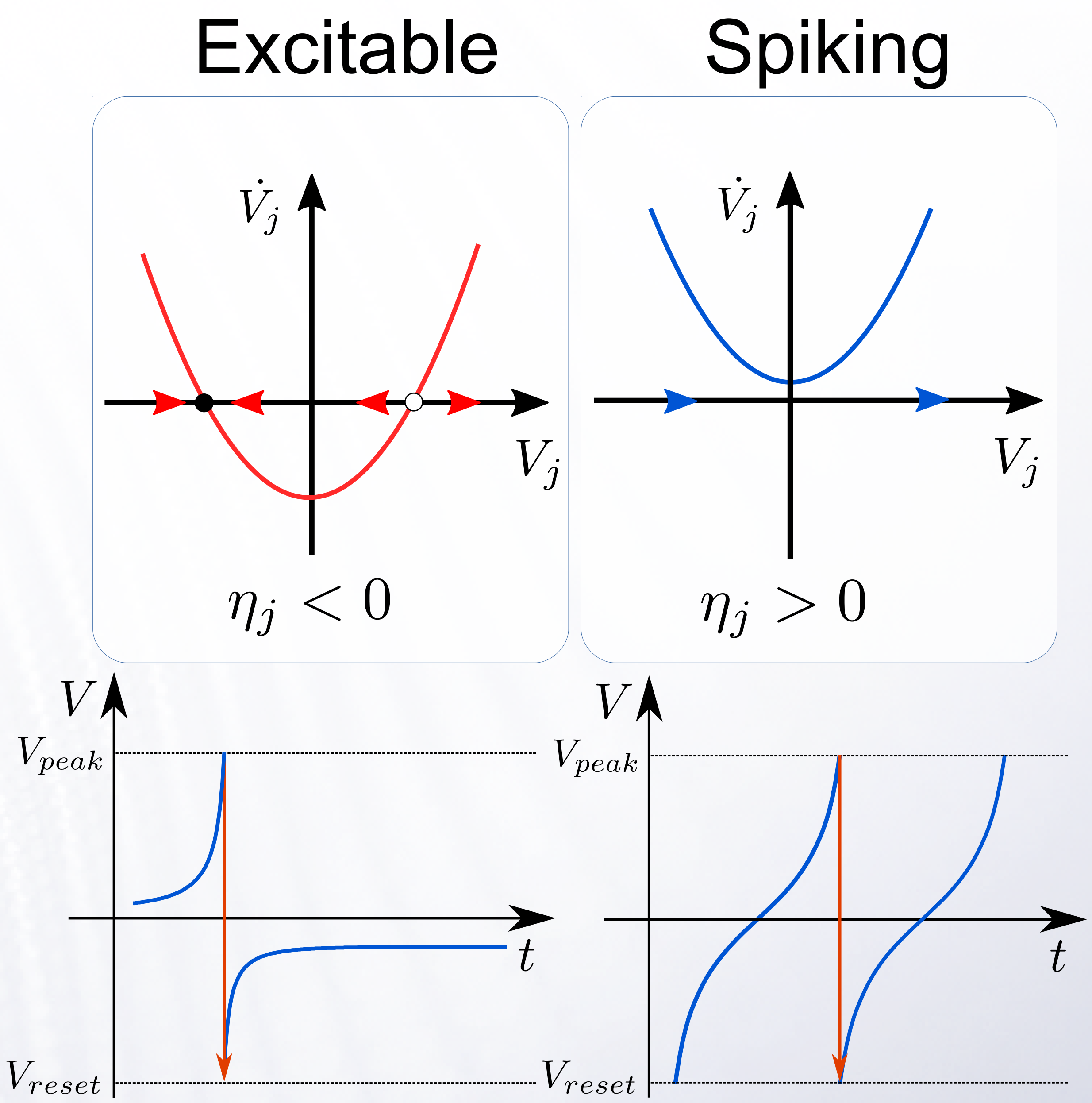
Quadratic integrate-and-fire neurons

Equations:

$$\dot{V}_j = V_j^2 + \eta_j$$

$$V_j \geq V_{peak} \qquad V_j = V_{reset}$$

$$V_{peak} = -V_{reset} \to \infty$$



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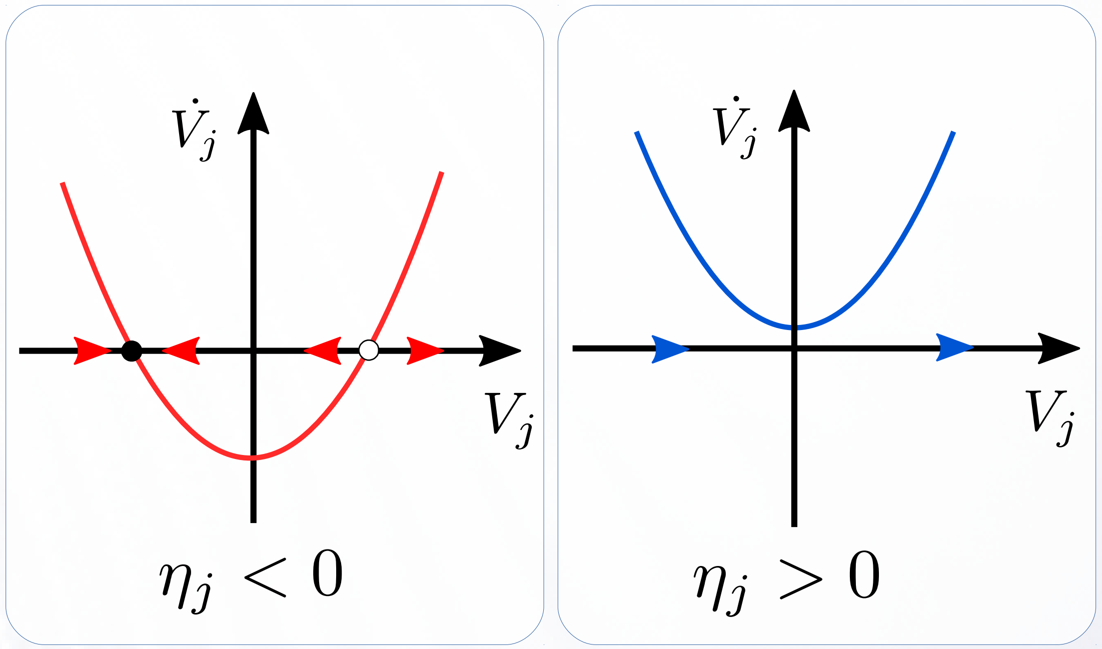
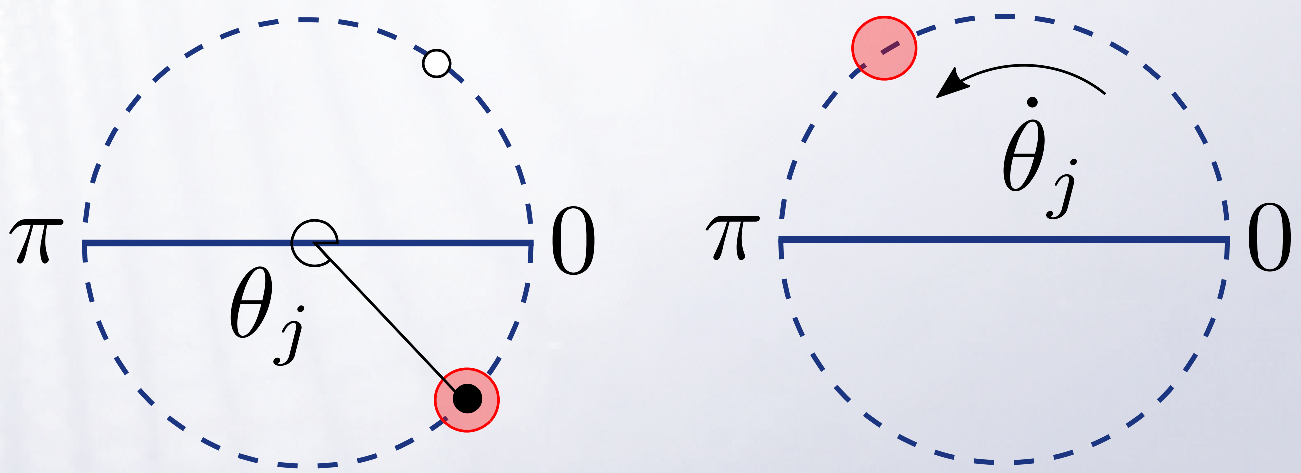
Excitable | Spiking



Theta representation

$$\theta_j = 2\arctan(V_j)$$

$$\dot{\theta}_j = (1 - \cos\theta_j) + (1 + \cos\theta_j)\eta_j$$



Interaction

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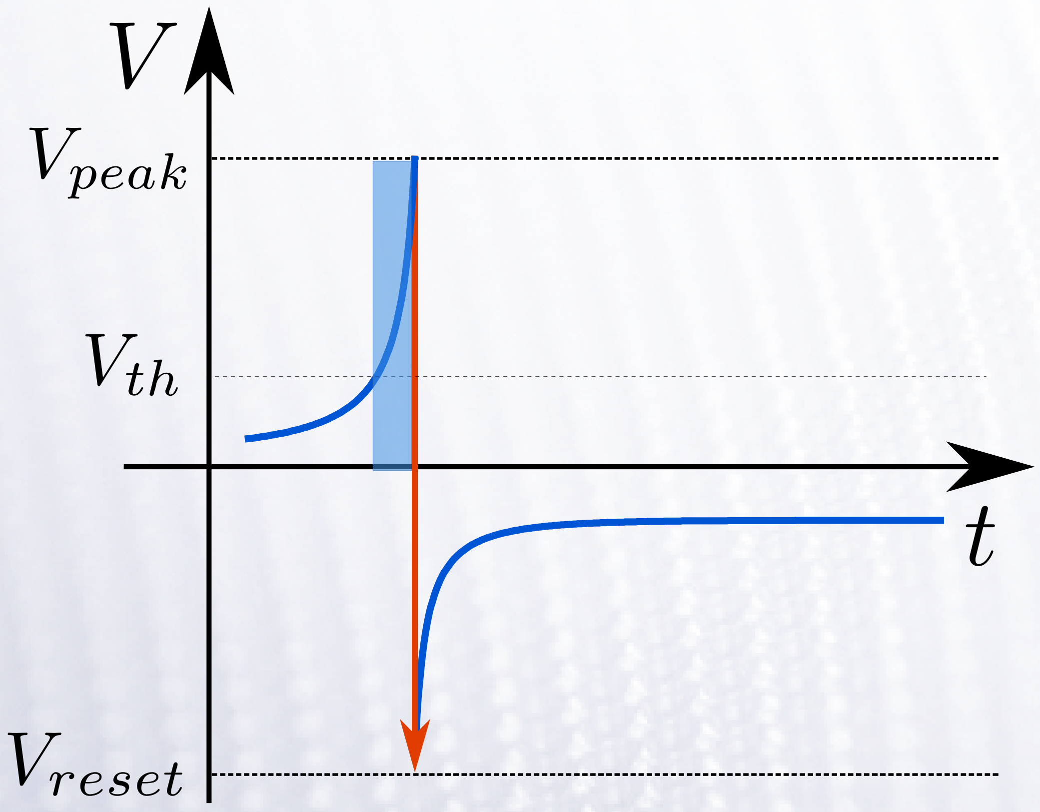
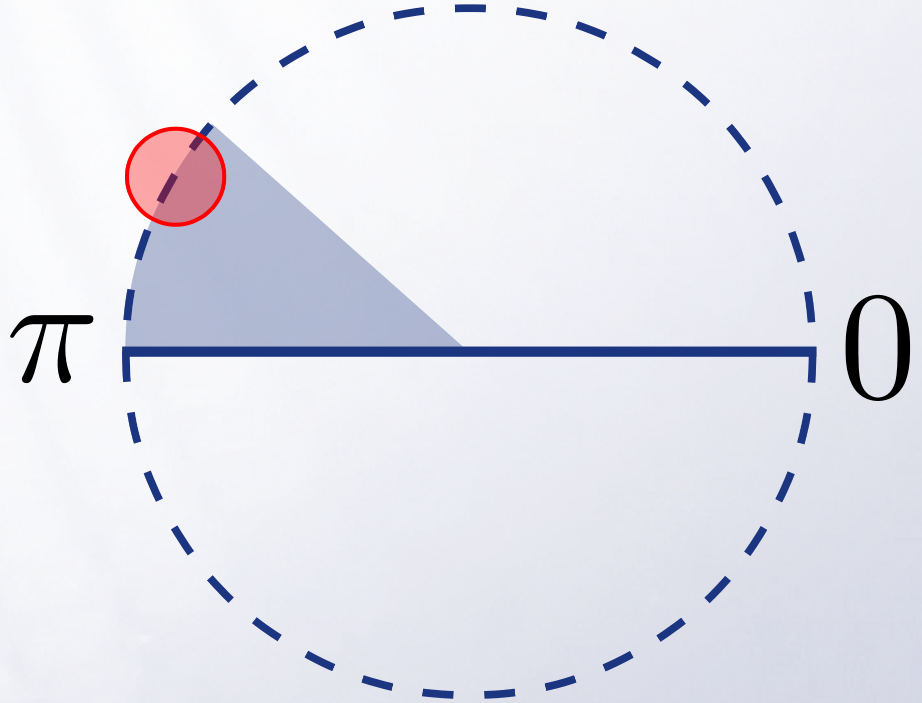
Neurons interact synaptically

Modeled by Heaviside function

$$I^{syn} = J\frac{V_{th}}{N}\sum_{l=1}^{N} H(V_l - V_{th})$$





Neuron effects other neurons only, when its potential exceed threshold value.

Macroscopic variables

Microscopic model

$$\dot{V}_j = V_j^2 + \eta_j + I^{syn}$$

$$V_{peak} = -V_{reset} \to \infty$$

$$I^{syn} = J \frac{V_{th}}{N} \sum_{l=1}^{N} H(V_l - V_{th})$$

Macroscopic variables:

- Mean membrane potential
- Firing rate

Infinite size network limit $N \to \infty$ enables analytical approach.

Continuity equation

Continuity density function: $\rho_k(V|\eta, t)dV$ number of neurons between V and $V + dV$.

Continuity equation

$$\frac{\partial}{\partial t}\rho_k = -\frac{\partial}{\partial V}\left[\rho_k\left\{V^2 + \eta + I^{syn}\right\}\right]$$

Trivial stationary solution:

$$\rho_k \propto (V^2 + \eta + I^{syn})^{-1}$$

Continuity equation

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Lorentzian ansatz

E. Montbrio, D. Pazo, A. Roxin , Phys. Rev. X 5, 021028 (2015)

$$\rho_k(V|\eta, t) = \frac{1}{\pi} \frac{x_k(\eta,t)}{[V - y_k(\eta,t)]^2 + x_k(\eta,t)^2}$$

$r_k(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} x_k(\eta, t) g(\eta) d\eta$ - population firing rate

$\mathrm{v}_k(t) = \int_{-\infty}^{+\infty} y_k(\eta, t) g(\eta) \mathrm{d}\eta$ - average potential

$g(\eta)$ distribution of parameter η

$$\begin{aligned}
\dot{x}_k(\eta, t) &= 2x_k(\eta, t) y_k(\eta, t), \\
\dot{y}_k(\eta, t) &= \eta - x_k^2(\eta, t) + y_k^2(\eta, t) + I^{syn}
\end{aligned}$$

Macroscopic equations

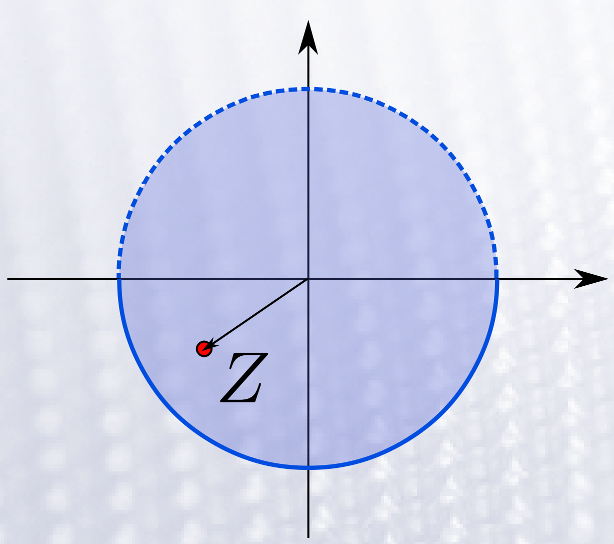
If external currents are distributed according to Lorentz function with width Δ and center $\bar{\eta}$. (Network consists both excitable and spiking neurons)

$$
\begin{aligned}
\dot{r}_k &= \Delta/\pi + 2 r_k v_k, \\
\dot{v}_k &= \bar{\eta} + v_k^2 - \pi^2 r_k^2 + I^{syn}
\end{aligned}
$$

Relation with Kuramoto order parameter:

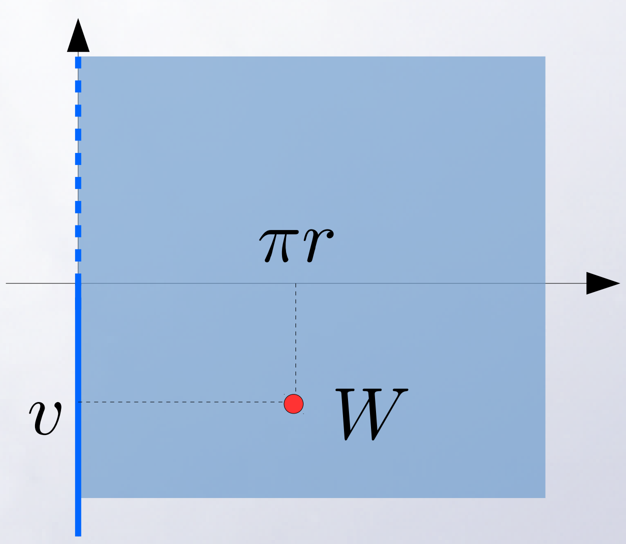
$$Z = \int \int \rho(V|\eta, t) g(\eta) e^{i\theta} d\eta d\theta$$

$$W = \pi r + iv$$



$$Z = \frac{1 - W^*}{1 + W^*}$$

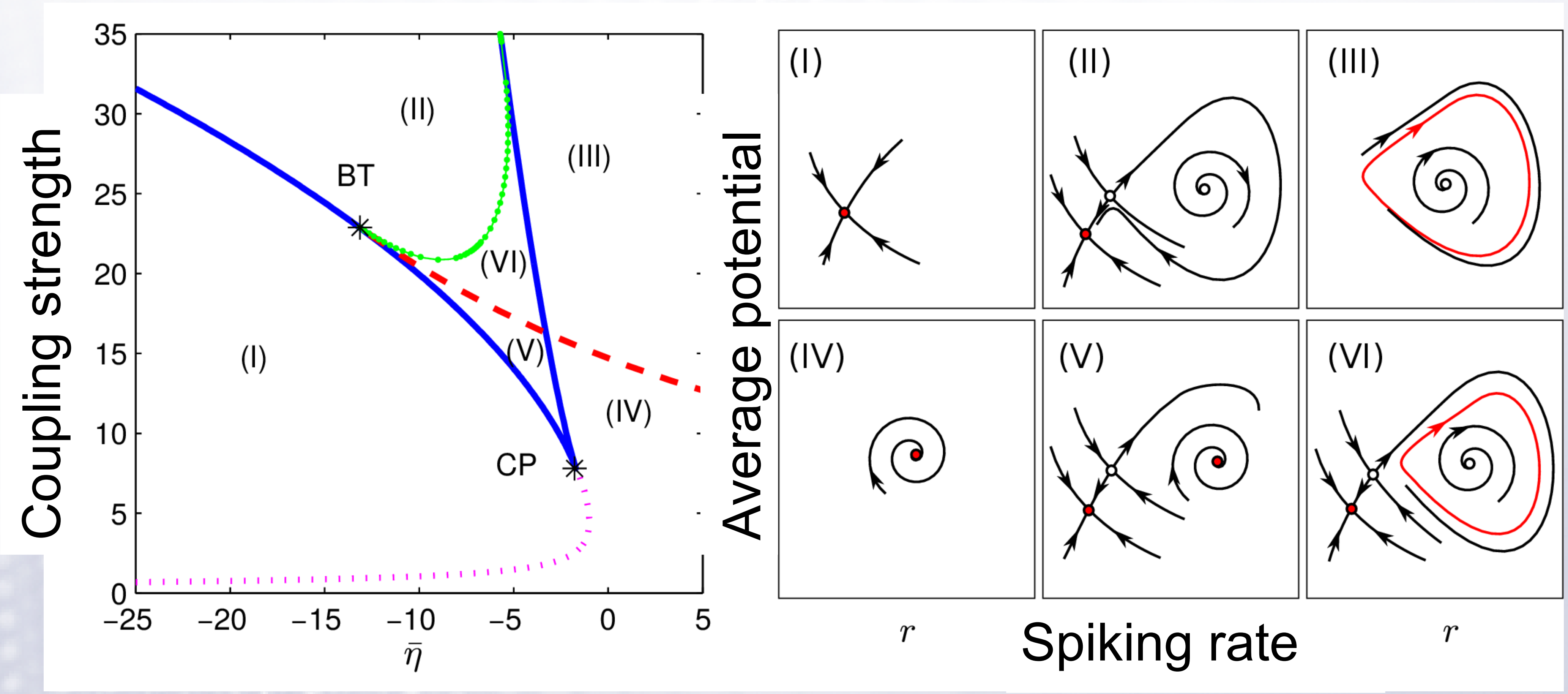
$$W = \frac{1 - Z^*}{1 + Z^*}$$



Macroscopic equations

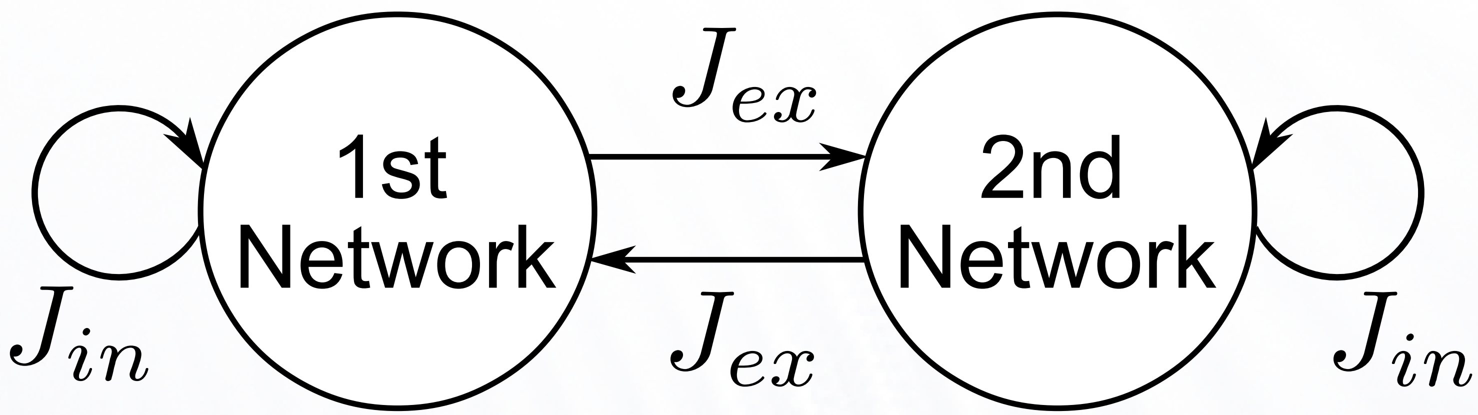
If external currents are distributed according to Lorentz function with width Δ and center $\bar{\eta}$

$$\begin{aligned} \dot{r}_k &= \Delta/\pi + 2 r_k v_k, \\ \dot{v}_k &= \bar{\eta} + v_k^2 - \pi^2 r_k^2 + I^{syn} \end{aligned}$$



Network

Identical populations



interactions

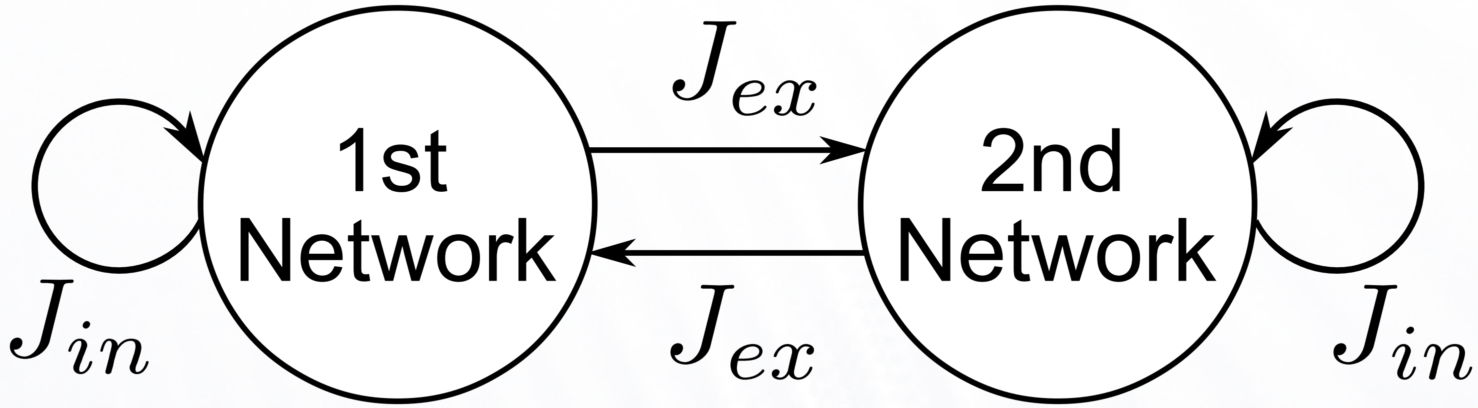
$$\dot{V}_{j,0} = V_{j,0}^2 + \eta_{j,0} + J_{in} S_0 + J_{ex} S_1$$
$$\dot{V}_{j,1} = V_{j,1}^2 + \eta_{j,1} + J_{in} S_1 + J_{ex} S_0$$

internal external

$$S_k = \frac{V_{th}}{N} \sum_{i=1}^{N} H(V_{i,k} - V_{th})$$

Network

Identical populations



interactions

$$\dot{V}_{j,0} = V_{j,0}^2 + \eta_{j,0} + J_{in} S_0 + J_{ex} S_1$$
$$\dot{V}_{j,1} = V_{j,1}^2 + \eta_{j,1} + J_{in} S_1 + J_{ex} S_0$$

internal external

Macroscopic equations:

$$\dot{r}_k = \Delta/\pi + 2 r_k v_k,$$
$$\dot{v}_k = \bar{\eta} + v_k^2 - \pi^2 r_k^2 + J_{in} S_k + J_{ex} S_{1-k}$$

$k = 0, 1$

Symmetric solutions

Transverse and longitudinal coordinates:

$$R = (r_0 - r_1)/2, \qquad Q = (r_0 + r_1)/2,$$
$$P = (v_0 - v_1)/2, \qquad M = (v_0 + v_1)/2$$

Symmetric solutions: $(0, 0, Q, M)$

Equations for *Q* and *M* are identical to eq. of a single population with a modified coupling strength $J = J_{in} + J_{ex}$ (equilibrium points and limit cycles)

$$\dot{Q} = \Delta/\pi + 2QM, \qquad \dot{r}_k = \Delta/\pi + 2r_k v_k,$$
$$\dot{M} = \bar{\eta} + M^2 - \pi^2 Q^2 + JV_{th}S. \qquad \dot{v}_k = \bar{\eta} + v_k^2 - \pi^2 r_k^2 + JV_{th}S$$

Symmetric solutions

Transverse and longitudinal coordinates:

$$R = (r_0 - r_1)/2, \qquad Q = (r_0 + r_1)/2,$$
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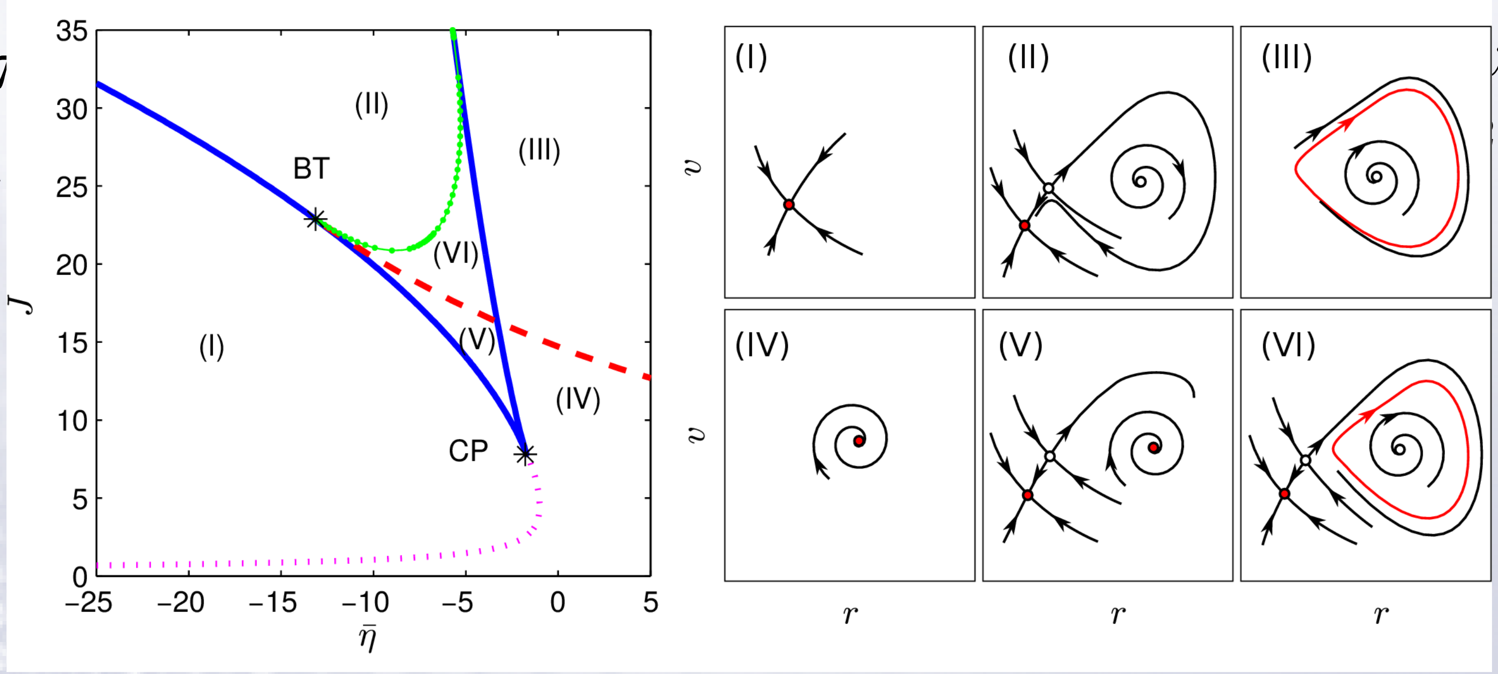
$$\dot{Q} = \Delta/\ldots \; \ldots v_k,$$
$$\dot{M} = \bar{\eta} + \ldots \; r_k^2 + J V_{th} S$$



Symmetric solutions

Transverse and longitudinal coordinates:

$$R = (r_0 - r_1)/2, \qquad Q = (r_0 + r_1)/2,$$
$$P = (v_0 - v_1)/2, \qquad M = (v_0 + v_1)/2$$

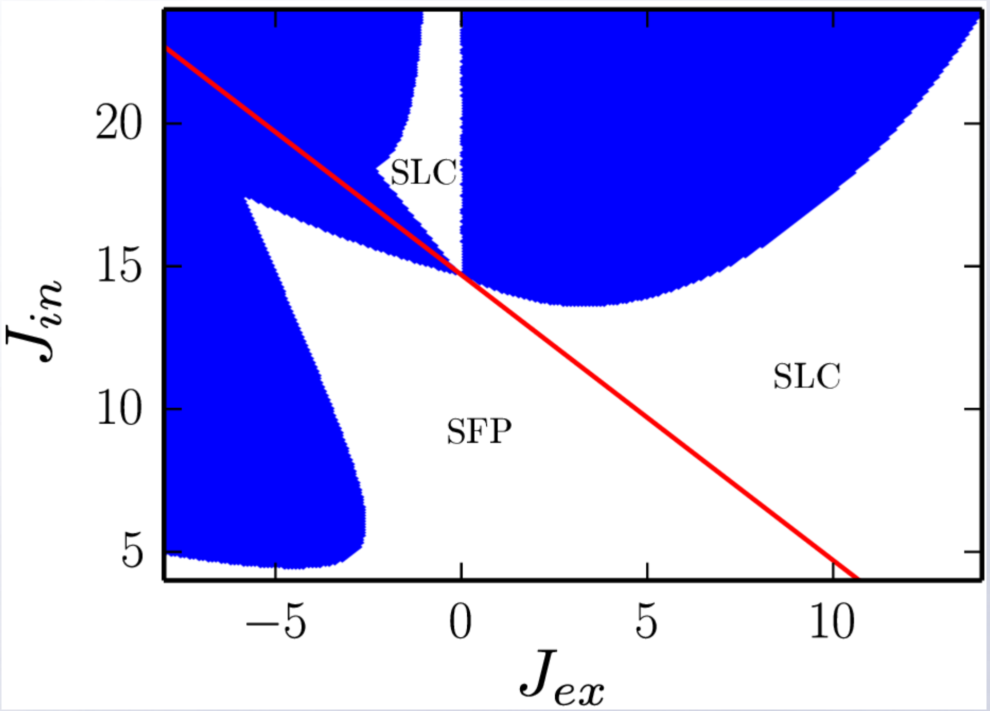
Symmetric solutions: $(0, 0, Q, M)$

Equations for *Q* and *M* are identical to eq. of a single population with a modified coupling strength $J=J_{in}+J_{ex}$ (equilibrium points and limit cycles)

$$\begin{pmatrix} \delta\dot{R} \\ \delta\dot{P} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} \delta R \\ \delta P \end{pmatrix}$$

$\boldsymbol{A}(t) = \text{const.}$ stationary points

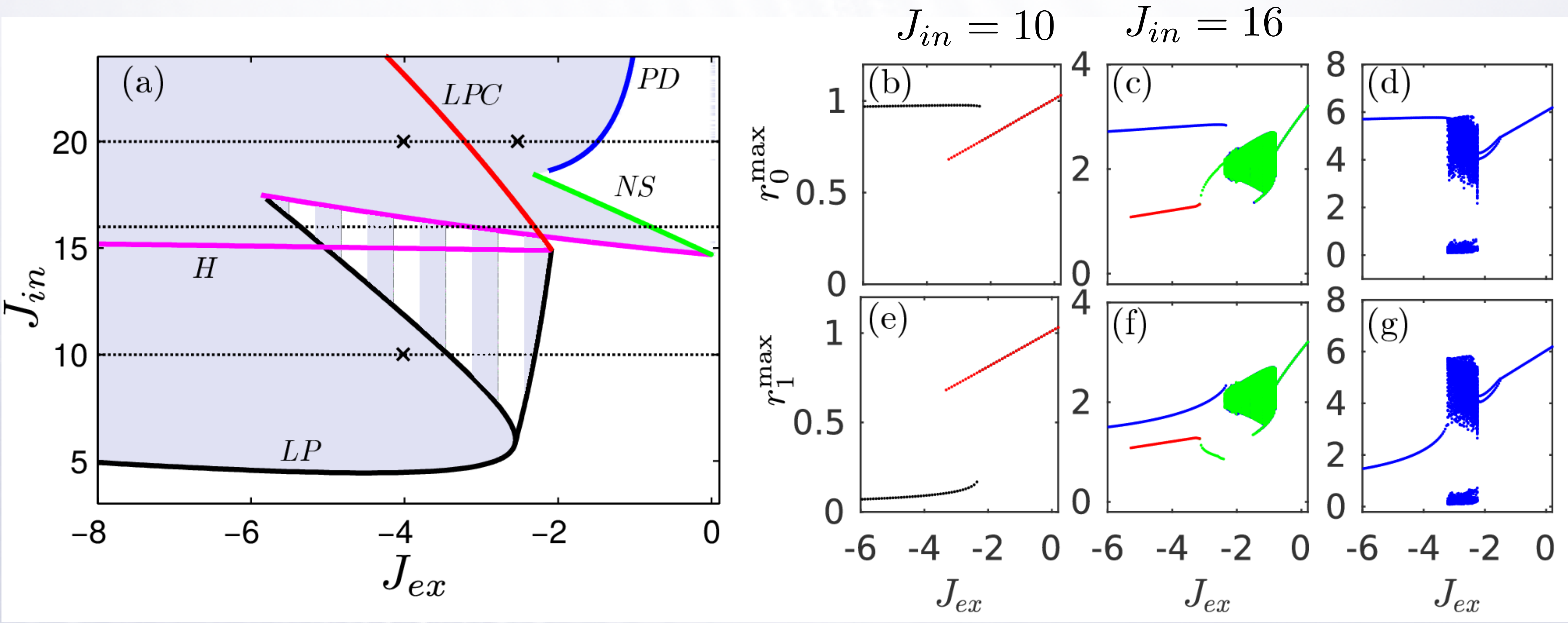
$\boldsymbol{A}(t) = \boldsymbol{A}(t+T)$ limit cycles



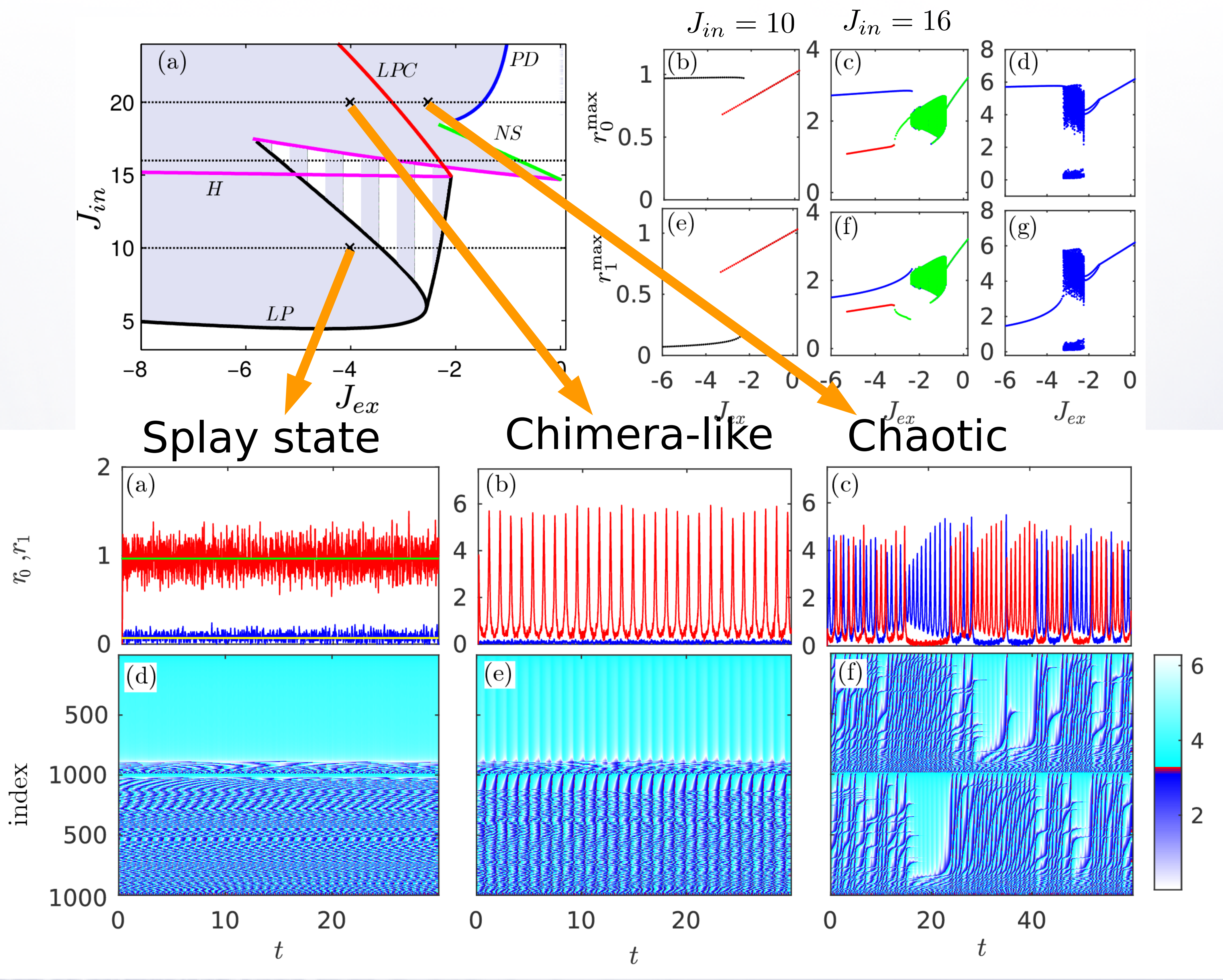
Non-symmetric solutions

Macroscopic equations:

$$\dot{r}_k = \Delta/\pi + 2r_k v_k, \qquad k = 0, 1$$
$$\dot{v}_k = \bar{\eta} + v_k^2 - \pi^2 r_k^2 + J_{in} S_k + J_{ex} S_{1-k}$$



Non-symmetric solutions



Non-symmetric solutions

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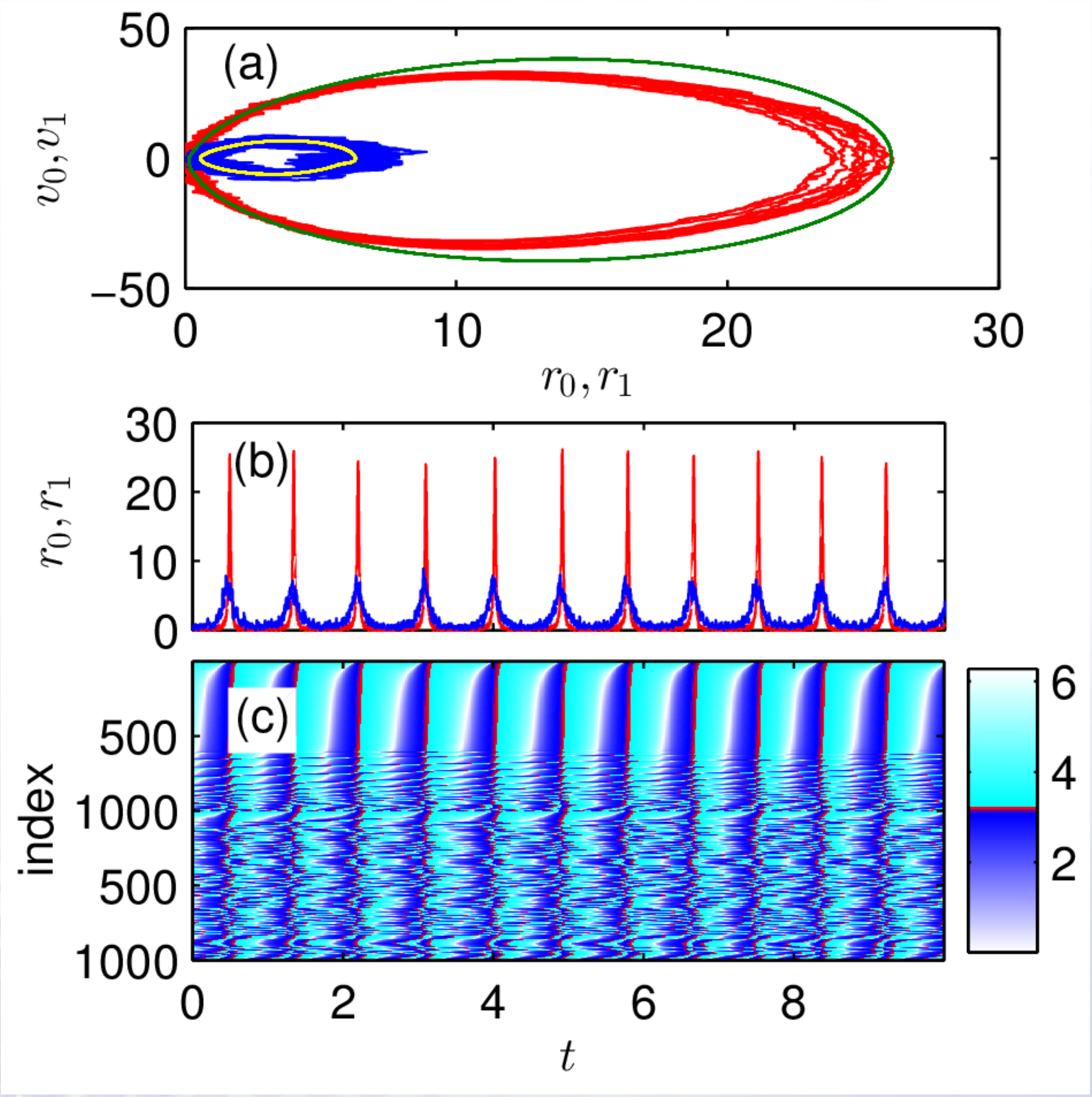
Chimera-like state for external excitatory coupling



Conclusions

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Competition of neural interactions within and between the populations may lead to a rich variety of nonsymetrical patterns, including splay state, antiphase periodic oscillations, chimera like states and chaotic oscillations as well as bistabilities between them.

I. Ratas and K. Pyrgas, *Symmetry breaking in two interacting populations of quadratic integrate-and-fire neurons*, Phys. Rev. E 96, 042212 (2017).

I. Ratas and K. Pyragas, *Macroscopic self-oscillations and aging transition in a network of synaptically coupled quadratic integrate-and-fire neurons*, Phys. Rev. E 94, 032215 (2016).

ACKNOWLEDGMENT

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