Desynchronization of oscillatory networks by intermittent delayed feedback control

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Outline

- Motivation
- Algorithm scheme
- Landau-Stuart oscillators desynchronisation
- Hodgkin-Huxley neurons desynchronisation
- Conclusions

Motivation

- Possible application for the deep brain stimulation
- Previously proposed desynchronisation methods

  P.A. Tass, Biol. Cybern., 89:81–88, 2003
  K. Pyragas, O.V. Popovich and P.A. Tass, Europhys. Lett., 80:40002, 2007
  N. Tukhlina, M. Rosenblum, A. Pikovsky, and J. Kurths, Phys. Rev. E, 75:011918, 2007
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- Drawbacks
  - It uses more than one electrode
  - Feedback is not protected from stimulation signal direct impact

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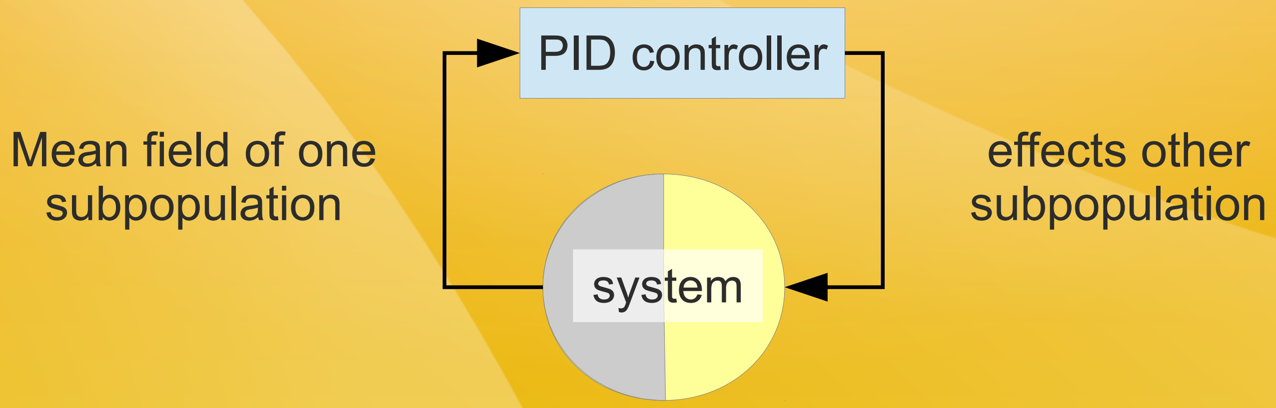
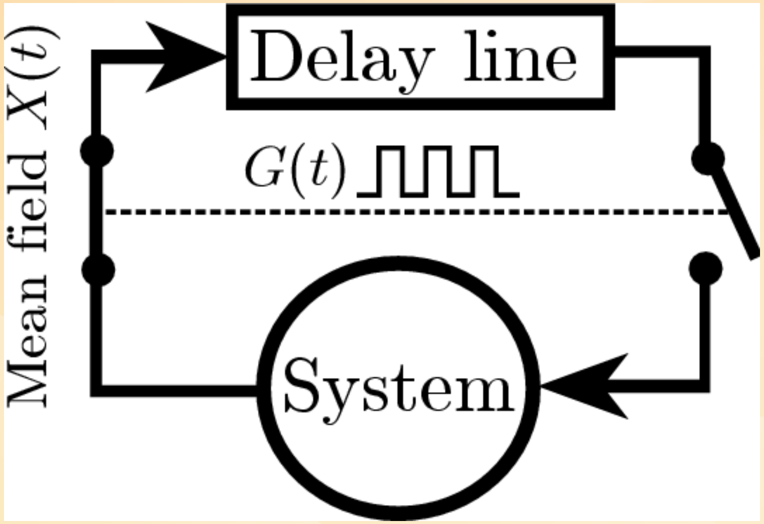


Algorithm scheme

Stage I



In the first stage, we measure and memorize the output of the control-free system.

Algorithm scheme

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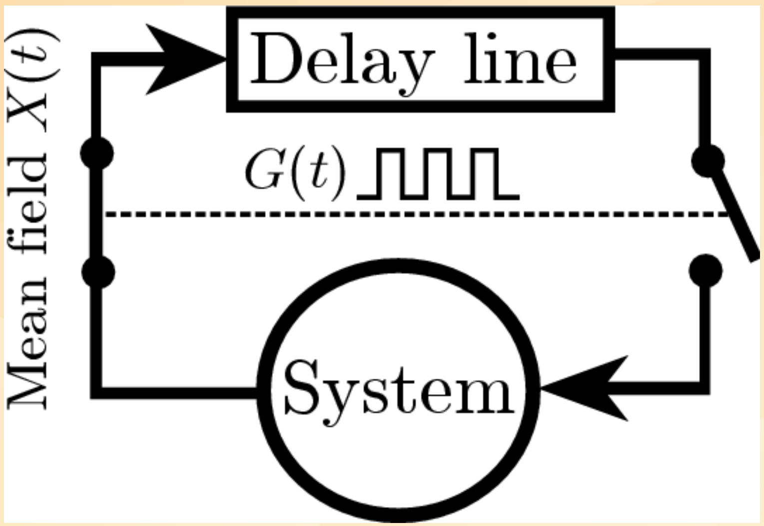

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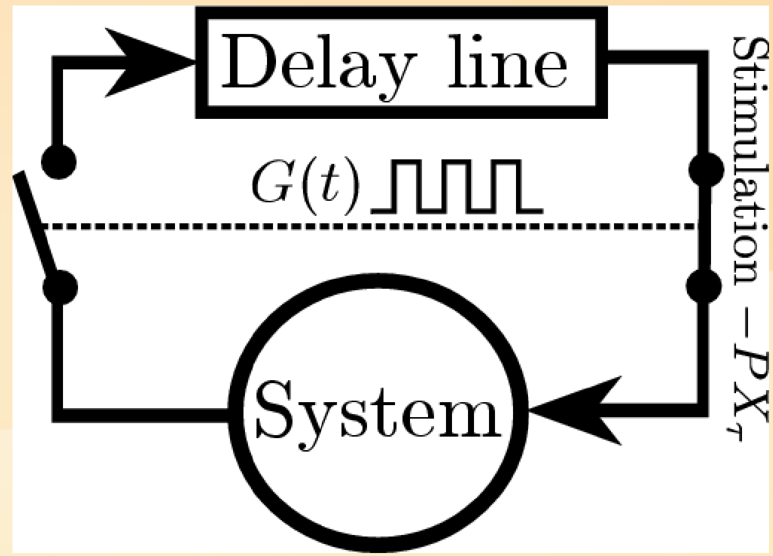
Stage II



In the second stage, we apply the feedback control using the memorized signal. Both stages take equal amount of time τ.

Desynchronisation of Landau-Stuart oscillators

$$\underbrace{\dot{z}_j = (i\omega_j + 1 - |z_j|^2)z_j}_{\text{oscillator}} + \underbrace{KZ}_{\text{coupling}} - \underbrace{PZ_\tau G(t)}_{\text{control}} \qquad (1)$$

Complex variable: $z_j = \rho_j e^{i\theta_j}$

Effect of the averaged field: $Z = \frac{1}{N}\sum_{j=1}^{N} z_j$

The feedback coefficient *P* is complex measure.

System synchronisation is defined by order parameter: $r = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$

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$|r| = 1$
synchronised state

$|r| = 0$
desynchronised state

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Object is to reset *r* to 0

$|r| = 0$

desynchronised state

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Equation for order parameter

Equations for oscillators

$$\dot{z}_j = \ldots$$

?

Equation for the order parameter *r*

$$\dot{r} = \ldots$$

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Equations for oscillators

$$\dot{z}_j = ...$$

Equation for the order parameter *r*

$$\dot{r} = ...$$

Equations for oscillators phases
(Kuramoto model)

$$\dot{\theta}_j = ...$$

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Equation for the order parameter *r*

$$\dot{r} = ...$$

Assumptions:

1. All oscillators have the same radius.
2. The number of oscillators is infinite i.e. continuous case.
   - Ott-Antonsen ansatz – infinite size coupled oscillators behave low dimensional dynamics
3. The intrinsic oscillators frequencies are distributed by the Lorentzian (with central frequency ω_0 and width Δ).

Edward Ott and Thomas M. Antonsen, Chaos, 18:037113, 2008

Desynchronisation of Landau-Stuart oscillators

$$\dot{r} = \frac{K}{2}(r - r^* r^2) + \frac{1}{2}G(t)(P^* r^2 r_\tau^* - P r_\tau) + i(\omega_0 + i\Delta)r \quad (2)$$

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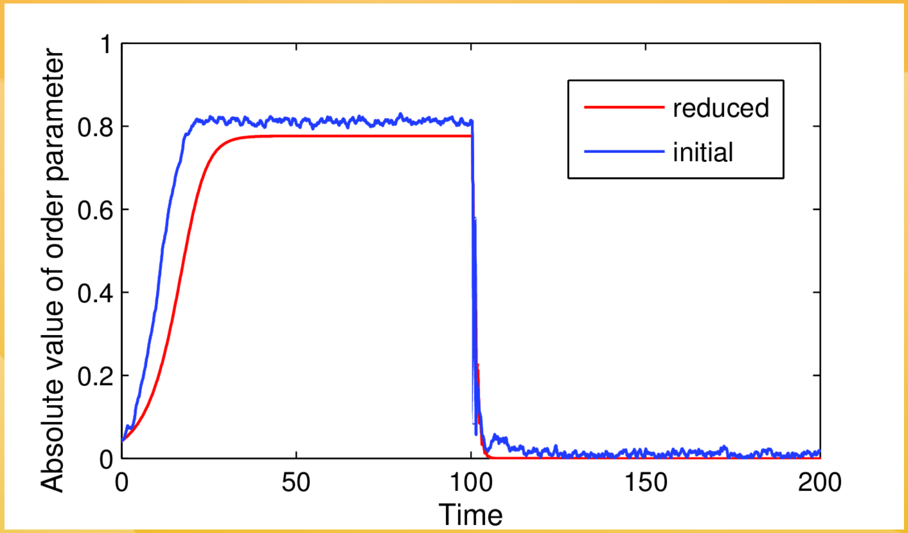
Solutions of Landau-Stuart equations and reduced equation for order parameter:



Parameters:
T=2
K=0.5
P=1.1+3.3 i
τ=0.4

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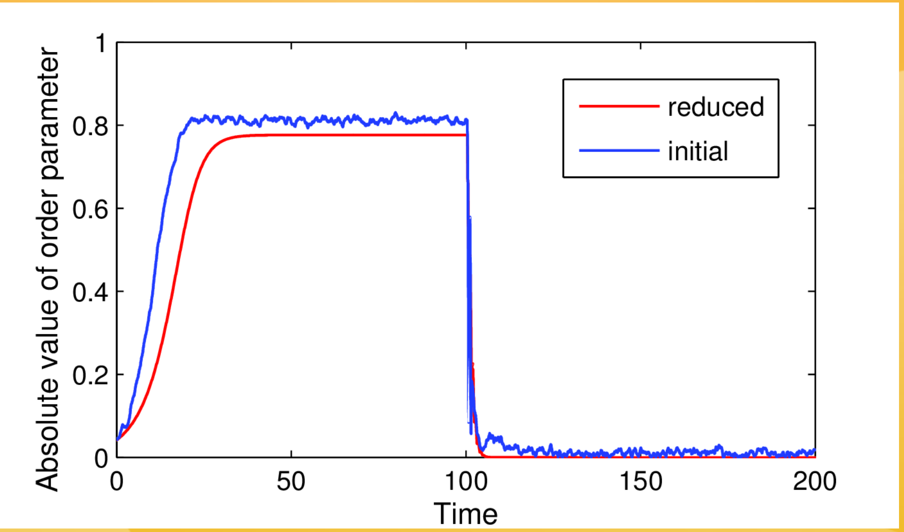

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Which *P* and *τ* desynchronise?

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Desynchronisation stability zones

Linearization reduces initial problem to unstable fixed point stabilisation:

$$\dot{x} = \lambda x - G(t)\bar{P}x_\tau \quad (3)$$

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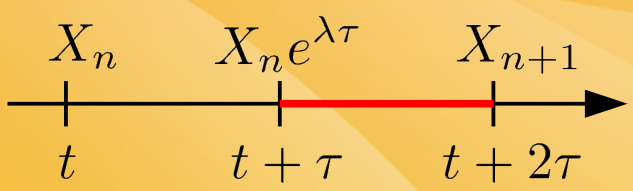
Zeroth point stability can be estimated studying one registration-stimulation period.



$$X_{n+1} = C(P,\tau)X_n$$

Stable when

$$|C(P,\tau)| < 1$$

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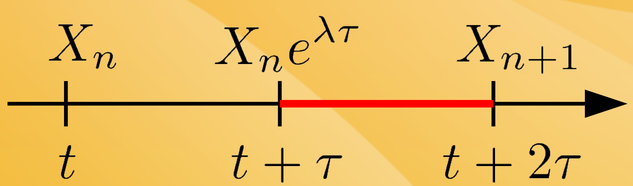

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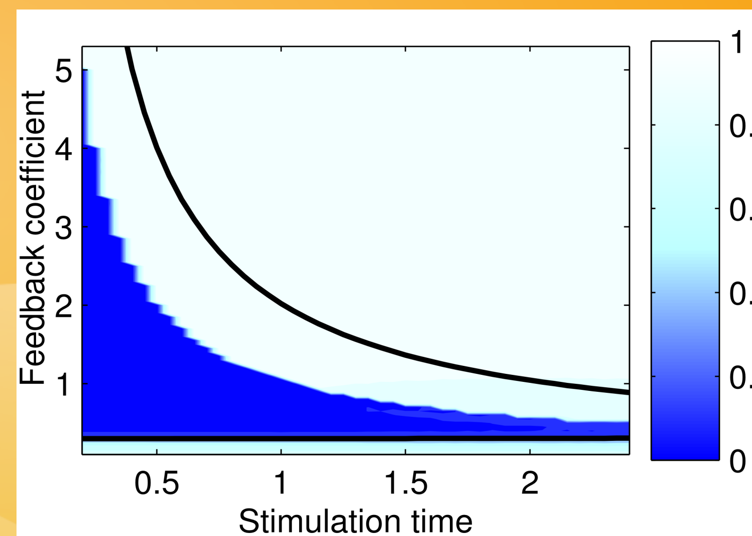

Color code shows oder parameter absolute value calculated from integration of original problem. According linear analysis the order parameter relax to zero between black lines.

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Synaptically coupled Hodgkin-Huxley(HH) neurons

Realistic neuron model:

$$\underbrace{C\dot{v}_k = F_1(v_k, m_k, h_k, n_k) + I_k}_{\text{Standart HH model}} \underbrace{- I_{syn,k}}_{\text{Coupling}} - \underbrace{PG(t)V_\tau}_{\text{Control}} \qquad (4)$$

v_k - neurons membrane potential
I_k - regulate neurons frequency
$I_{syn,k}$ - synaptic current- makes synchronisation

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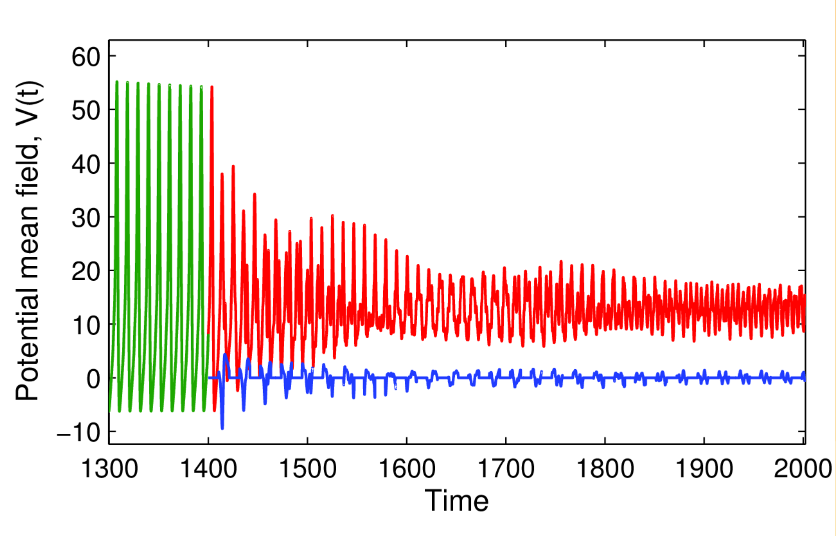


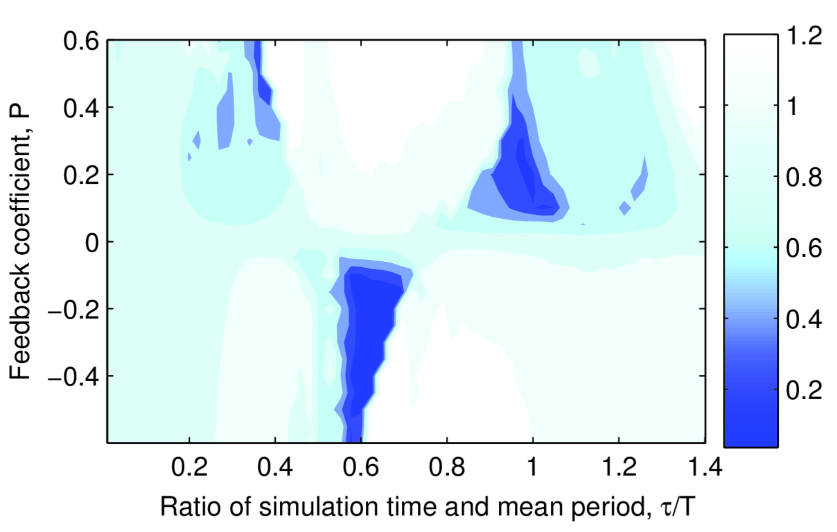
Solution of HH equations. Green- sync state (withouth algorithm), red- algorithm is on, blue line represent stimulation current.

Desyncronisation parameter is defined as ratio between variance of mean field when stimulation is on and free system:

$$S = \sqrt{\frac{\mathrm{Var}\left(V_{stim}\right)}{\mathrm{Var}\left(V_{free}\right)}}$$

- smaller is better

Depicted in colors.



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Conclusions

- Separation of the registration and stimulation stages in time allow implement algorithm with one electrode and avoid stimulation direct influence to feedback signal;
- Analytical estimations and numerical simulations confirm that the algorithm desynchronise globally coupled Landau-Stuart oscillators and synaptically coupled Hodgkin-Huxley neurons.

Acknowledgments
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