Application of act and wait control to oscillatory network desynchronization

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Outline

- Motivation
- Algorithm scheme
- Landau-Stuart oscillators desynchronisation
- Hodgkin-Huxley neurons desynchronisation
- Conclusions

Motivation

- Pathological synchronization - symptoms of neurological diseases
- Desynchronization methods:

  I) open loop (e.g. coordinates reset) – energetically inefficient

  II) closed loop (e.g. PID, delayed feedback) – uses more than one electrode and/or feedback is not protected from stimulation signal direct impact

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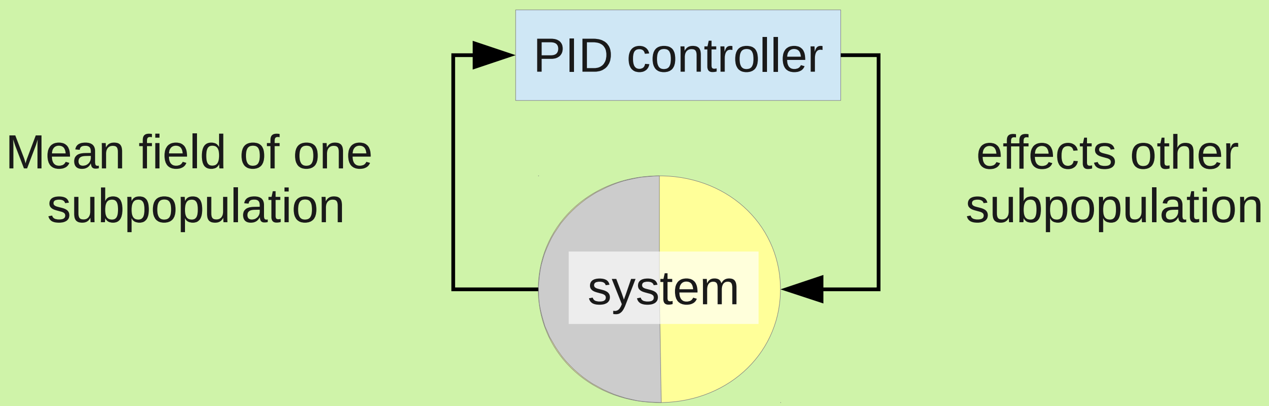
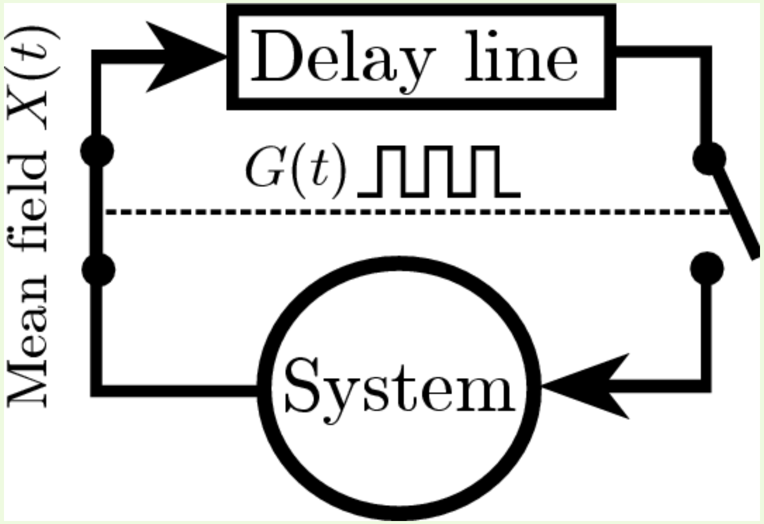


K. Pyragas, O.V. Popovych, P. A. Tass, EPL, **80** 40002 (2007)

Algorithm scheme

Stage I

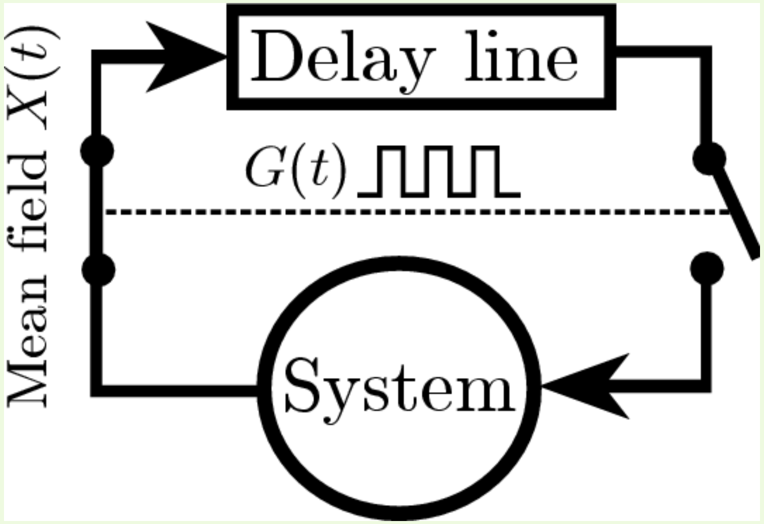


In the first stage, we measure and memorize the output of the control-free system.

Stage II

Algorithm scheme

Stage I



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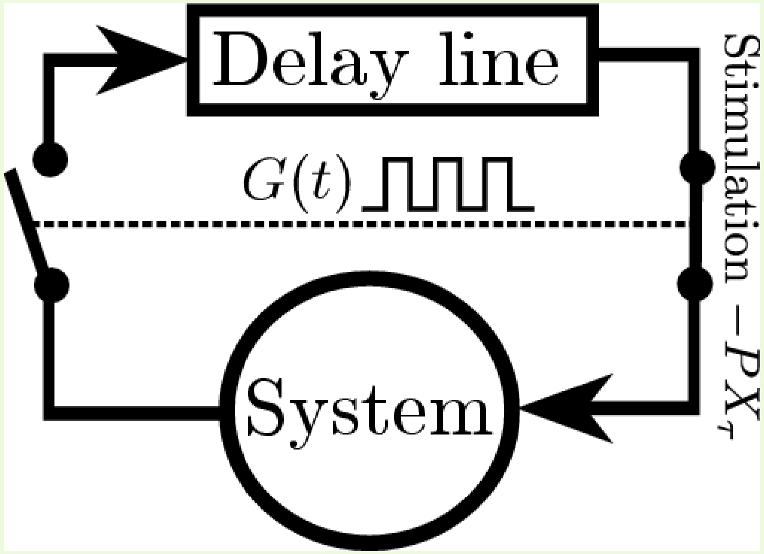

In the second stage, we apply the feedback control using the memorized signal. Both stages take equal amount of time τ.

Desynchronization of Landau-Stuart oscillators

$$\underbrace{\dot{z}_j = (i\omega_j + 1 - |z_j|^2)z_j}_{\text{oscillator}} + \underbrace{KZ}_{\text{coupling}} - \underbrace{PZ_\tau G(t)}_{\text{control}}$$

Complex variable: $z_j = \rho_j e^{i\theta_j}$

Effect of the averaged field: $Z = \frac{1}{N}\sum_{j=1}^{N} z_j$

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System synchronization is defined by the ***order parameter***:

$$r = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

$|r| = 1$
synchronised state

$|r| = 0$
desynchronised state

Object is to reset *r* to 0

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Equation for order parameter

?

Equations for oscillators

$$\dot{z}_j = \ldots$$

Equation for the order parameter *r*

$$\dot{r} = \ldots$$

Equations for oscillators phases
(Kuramoto model)

$$\dot{\theta}_j = \ldots$$

Assumptions:

1. All oscillators have the same radius.
2. The number of oscillators is infinite i.e. continuous case.
3. The intrinsic oscillators frequencies are distributed by the Lorentzian (with central frequency ω_0 and width Δ).

Ott-Antonsen ansatz – infinite size coupled oscillators behave low dimensional dynamics

Edward Ott and Thomas M. Antonsen, Chaos, 18:037113, 2008

Equation for order parameter

$$\dot{r} = \frac{K}{2}\left(r - r^* r^2\right) + i(\omega_0 + i\Delta)r + \underline{\frac{1}{2}G(t)\left(P^* r^2 r_\tau^* - P r_\tau\right)}$$

From control

Fixed point $r = x + \imath y = 0 + \imath 0$ exist.

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Linearization reduces initial problem to unstable fixed point stabilization:

$$\dot{x} = \lambda x - G(t)\bar{P}x_\tau$$

Zeroth point stability can be estimated studying one registration-stimulation period.

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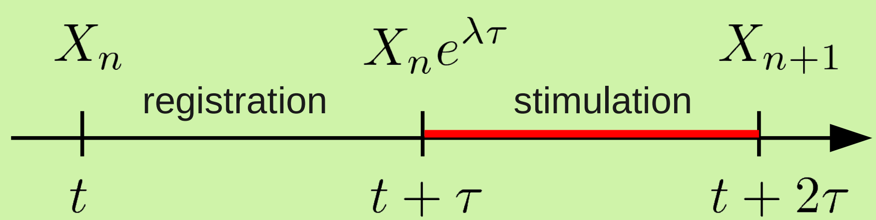
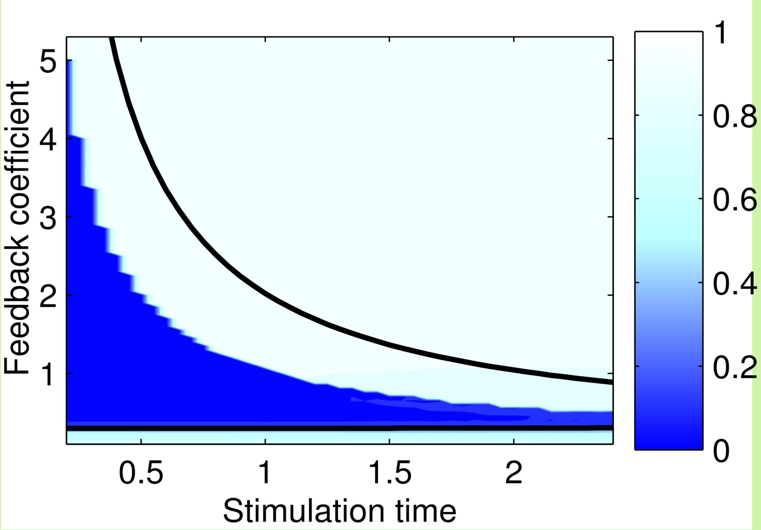
$$X_{n+1} = C(P,\tau)X_n$$



Stable, when

$$|C(P,\tau)| < 1$$

Desynchronization stability zones



Color code shows oder parameter absolute value calculated from integration of original problem. According linear analysis the order parameter relax to zero between black lines.

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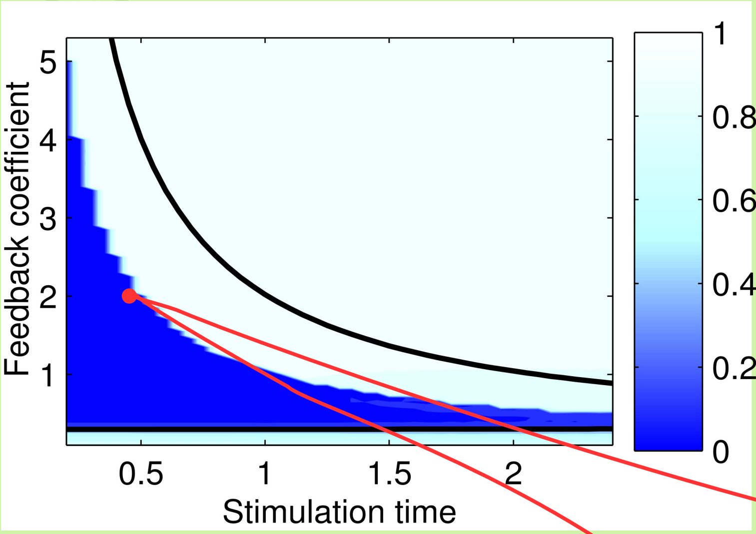

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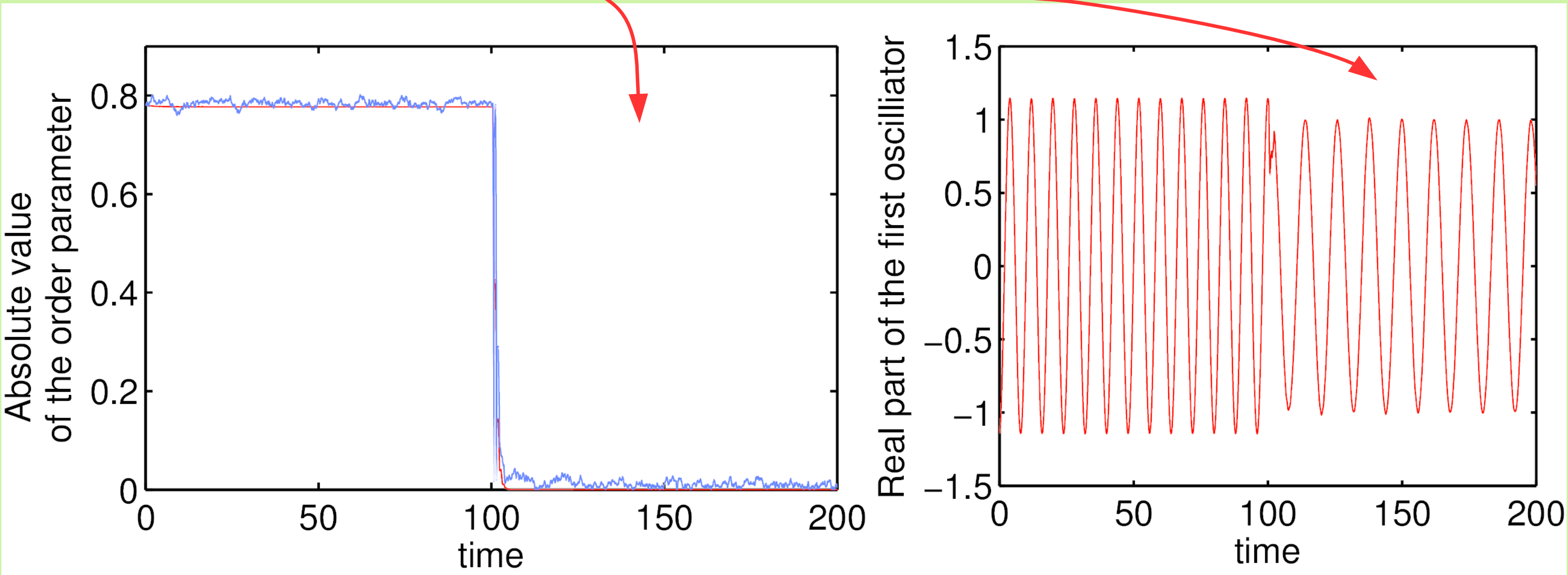

Parameters:

$\Delta = 0.1$

$\omega_0 = 0.25\pi$

$\bar{P} = 2$

$\tau = 0.4$

Landau-Stuart oscillators coupled through real parts

Equation form is similar to neurons equations:

$$\underbrace{\dot{z}_j = (i\omega_j + 1 - |z_j|^2)z_j}_{\text{oscillator}} + \underbrace{K\Re(Z)}_{\text{coupling}} - \underbrace{P\Re(Z_\tau)G(t)}_{\text{control}}$$

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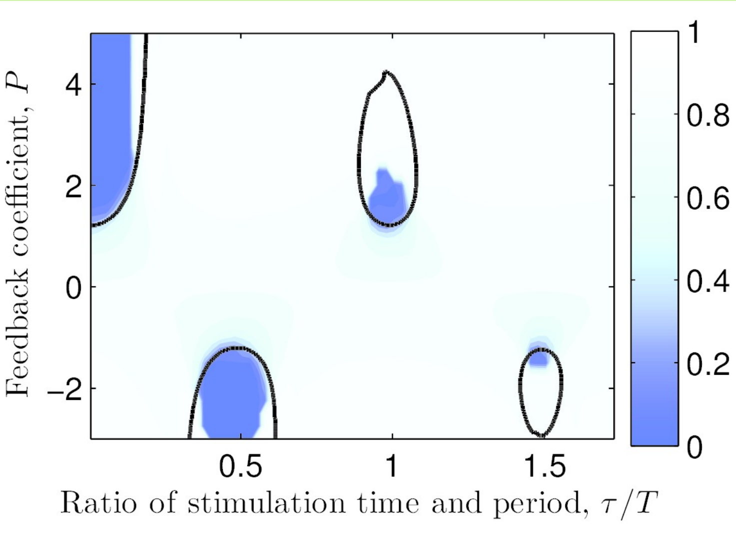
Stability zones of control
(color code)



Desynchronization will be possible, when

$$P > 0, \quad \tau/T = n$$
$$P < 0, \quad \tau/T = (n+1)/2$$

Here n is natural number.

On the other hand desynchronization regions with large n will be sufficiently small for practical use.

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Synaptically coupled Hodgkin-Huxley(HH) neurons

Realistic neuron model:

$$\underbrace{C\dot{v}_k = F_1(v_k, m_k, h_k, n_k) + I_k}_{\text{Standart HH model}} \underbrace{- I_{syn,k}}_{\text{Coupling}} \underbrace{- PG(t)V_\tau}_{\text{Control}}$$

v_k - neurons membrane potential
I_k - regulate neurons frequency
$I_{syn,k}$ - synaptic current- synchronize system
V_τ - delayed mean field

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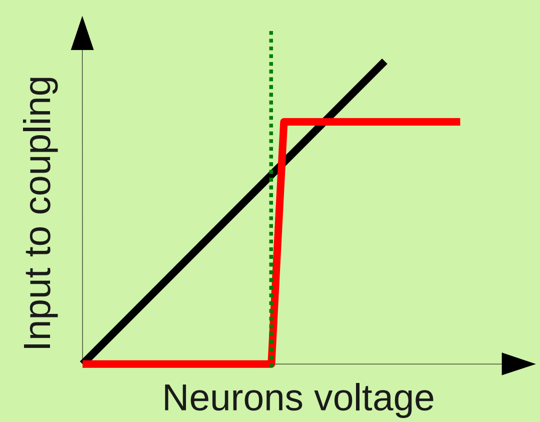
Difference between synaptic and mean field coupling



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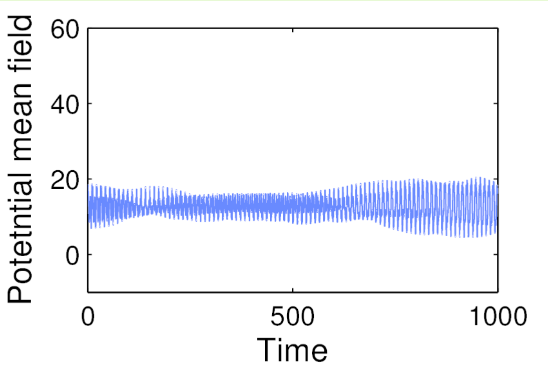
Synaptically coupled Hodgkin-Huxley(HH) neurons

- How to estimate synchronization in HH system?

  Highly synchronized system shows huge variations of mean field

Without coupling



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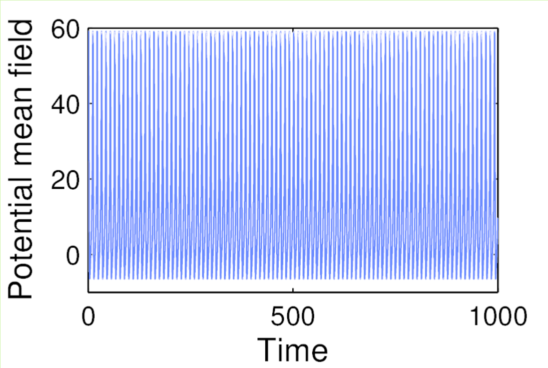

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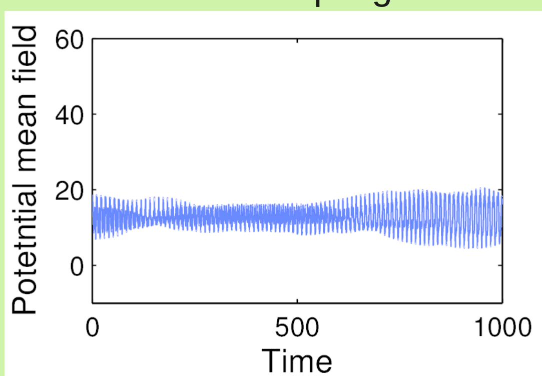

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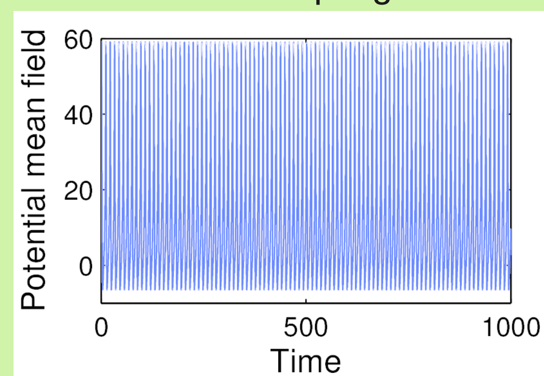


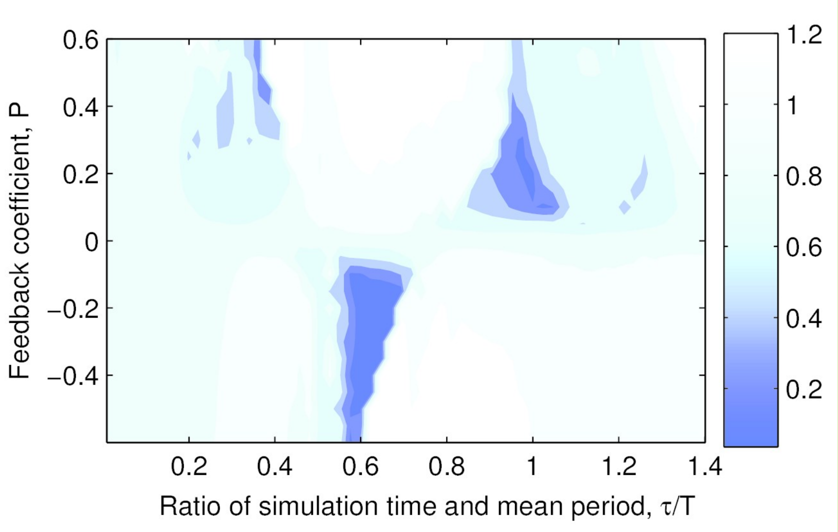
- Desyncronization parameter is defined as ratio between variance of mean field when stimulation is on and free system:

$$S = \sqrt{\frac{\mathrm{Var}\,(V_{stim})}{\mathrm{Var}\,(V_{free})}}$$ - smaller is better

M. Rosenblum, N. Tukhlina, A. Pikovsky, and L. Cimponeriu, Int. J. Bifurcat. Chaos **7**, 1989 (2006)

Synaptically coupled Hodgkin-Huxley(HH) neurons

Numerically estimated synchronization parameter S:



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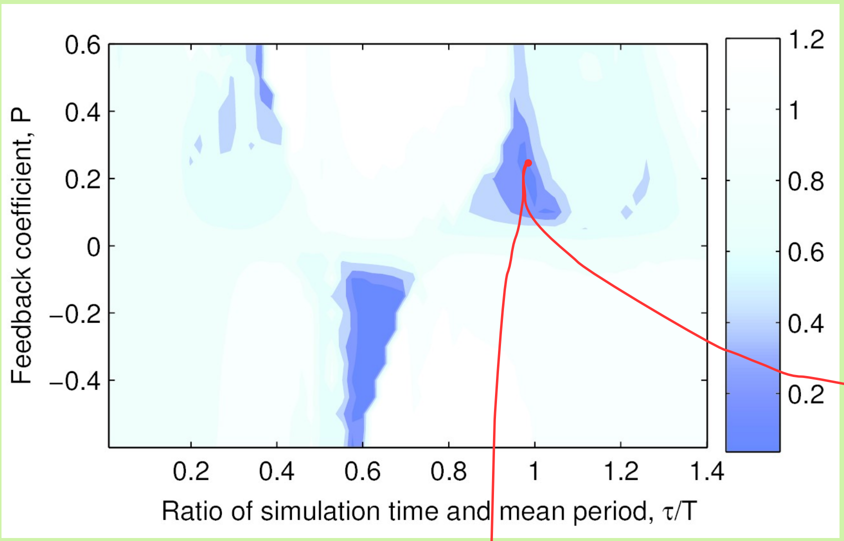

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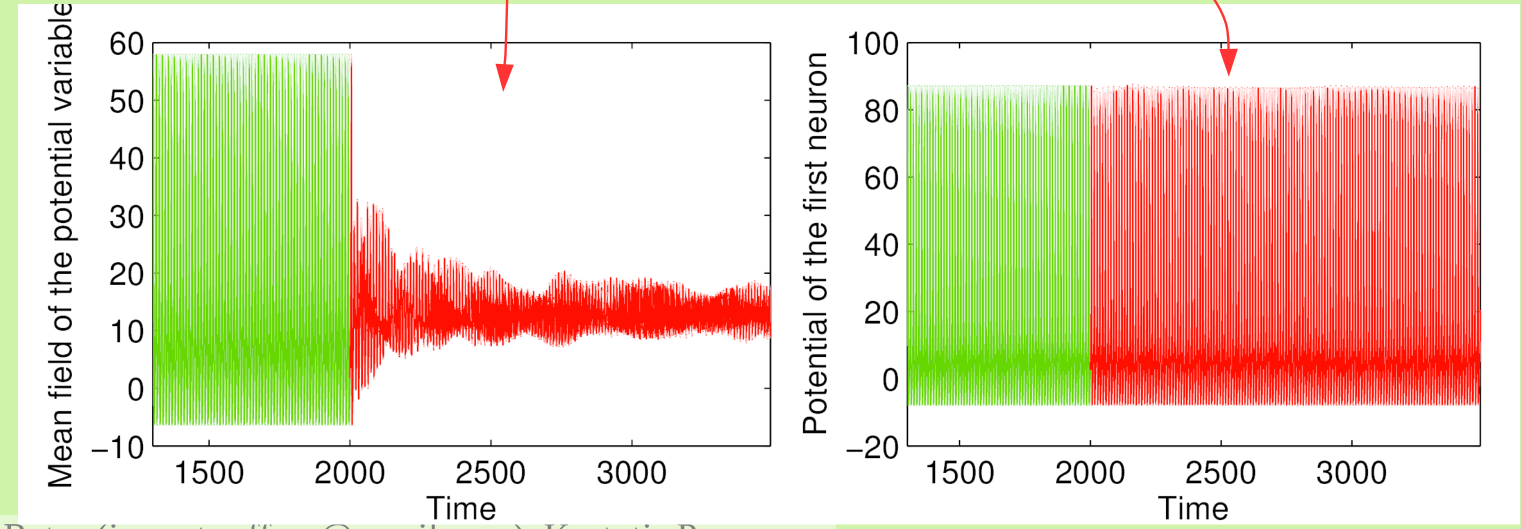

Parameters:

$P = 0.23$

$\tau = 10.5$

Conclusions

- Separation of the registration and stimulation stages in time allows us to implement algorithm with one electrode and avoid an influence of stimulation electrode to feedback signal;
- Analytical estimations and numerical simulations confirm that the act and wait algorithm can efficiently desynchronize globally coupled Landau-Stuart oscillators and synaptically coupled Hodgkin-Huxley neurons.

Acknowledgments
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Thank you for attention!

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