Noise-induced macroscopic oscillations in a network of synaptically coupled quadratic integrate-and-fire neurons

Irmantas Ratas (irmantas.ratas@ftmc.lt), Kestutis Pyragas

Center for Physical Sciences and Technology, Sauletekio av. 3, Vilnius, Lithuania

Results

We analyzed the dynamics of a large network of globally coupled quadratic integrate-and-fire neurons subjected to independent local noise [1]. The interaction between neurons is determined by synaptic pulses of a finite width. We found three different network modes: two of them are stable equilibrium states with low and high synaptic activity, and the third is a mode of limit-cycle oscillations. The oscillations mode is the most interesting as the initially quenched neurons are excited by noise, and their spikes are synchronized due to the interaction.

Model

We use quadratic integrate-and-fire neuron (QIF):

$$\dot{V}_j = V_j^2 + \underbrace{\eta_j}_{1} + \underbrace{S}_{2} + \underbrace{\sigma\xi_j(t)}_{3},$$

When V_j reaches V_{peak} it is reseted to V_{reset}. We choose $V_{\text{peak}} = -V_{\text{reset}} = \infty$.

1 Internal neuron's parameter determining, if it is in the oscillatory ($\eta_j > 0$) or in the excitable ($\eta_j < 0$) regime.

2 Synaptic all-to-all coupling.

3 Local independent Gaussian noise.

QIF is related to theta neuron model through transformation: $\theta_j = 2\arctan(V_j)$, which will be used in further analysis.

Steps of analysis

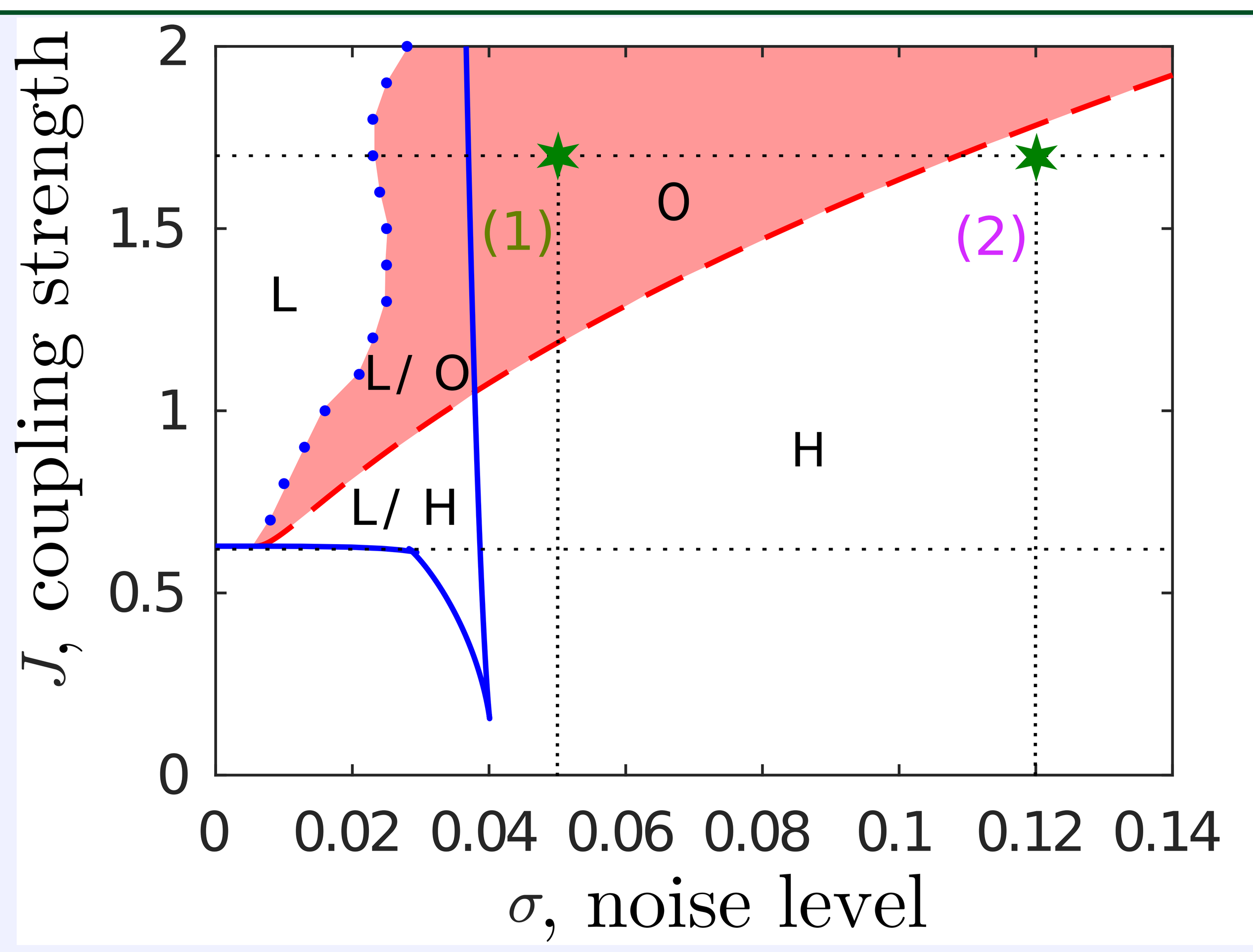
To achieve analytical insights, we investigate the thermodynamic limit of the network, i.e. infinite number of neurons. For such case we can:

1. To write the Fokker-Planck (FP) equation;
2. To expand FP equation into infinite number of equations for Kuramoto-Daido order parameters:
   $Z_n = \frac{1}{N}\sum_{j=1}^{N}\exp(in\theta_j)$;
3. To relate order parameters Z_n with cumulants κ_n by the use of moment generating function [2].

The introduction of cumulants enables us to use perturbation theory and reduce the initial problem with infinite number of differential equations to problem with small number m with an accuracy of $|\kappa_m| \propto \sigma^{2(m-1)}$.

Two cumulant case

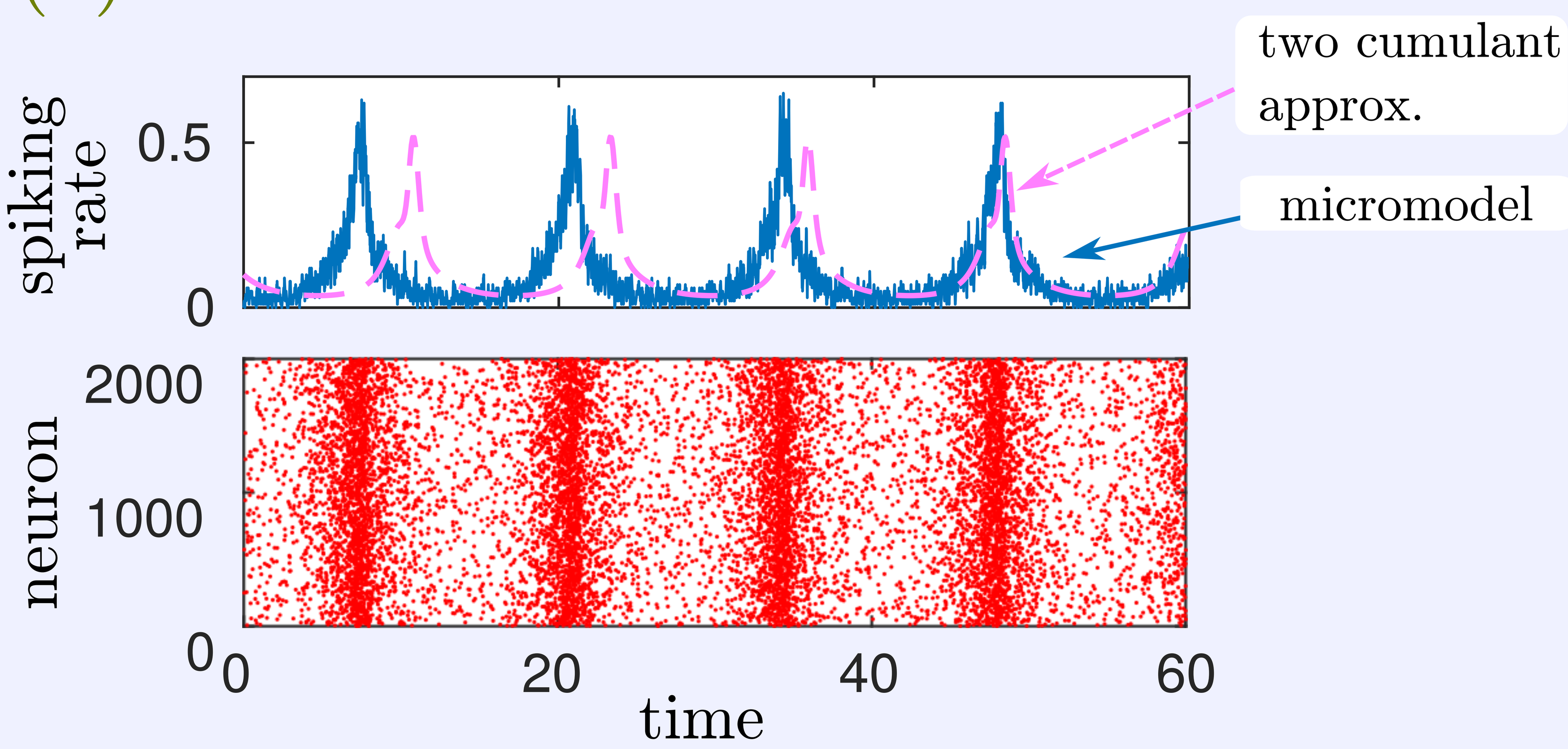
We truncate infinite chain of cumulant equations at the second cumulant, therefore we have two complex differiantial equations, which can be investigated by bifurcation analysis (MATCONT).



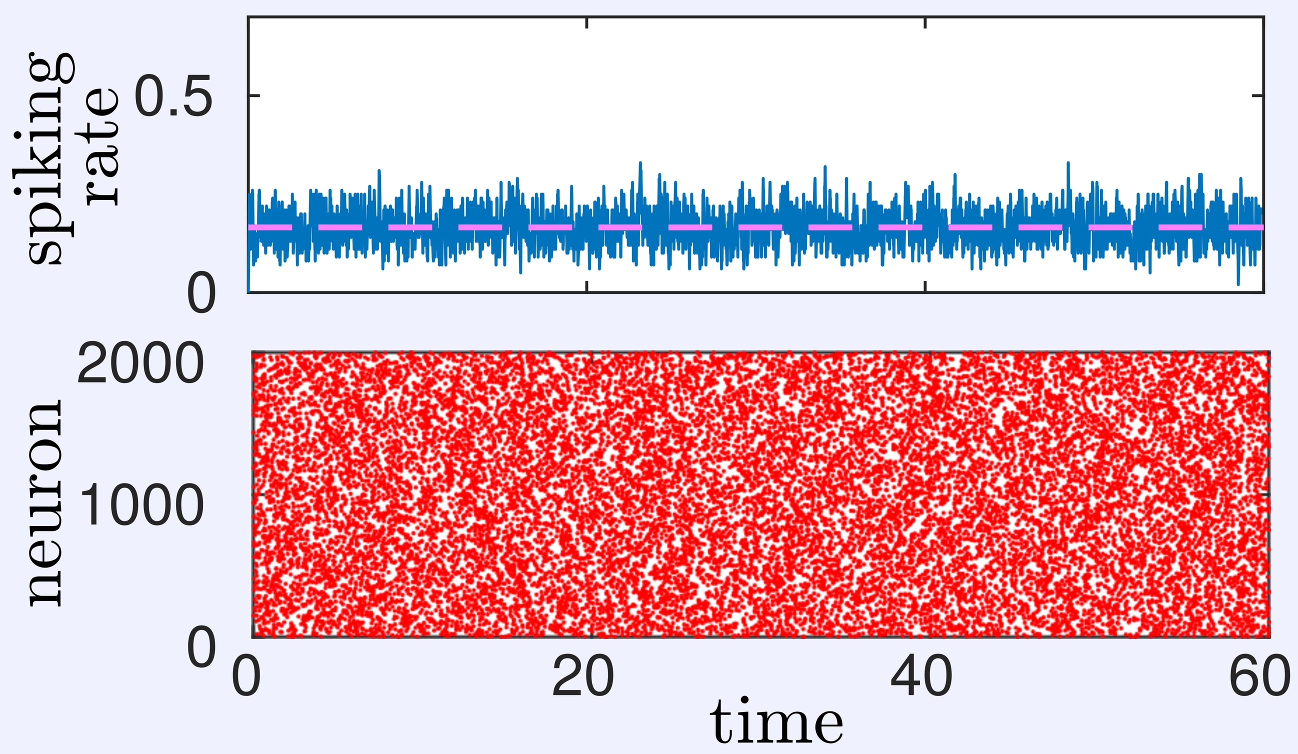
L - area of low synaptic activity (all neurons are quenched);
H - area of high synaptic activity (neurons are spiking incoherently);
O - area of macroscopic oscillations (neurons are spiking in a synchronized manner).

Comparison with finite network

(1) Noise induced oscillations:



(2) High synaptic activity case:



Conclusions

- Two-cumulant approximation reasonably good describes macroscopic behaviour of QIF network under effect of small local noise.
- Noise can induce synchronous oscillations in the system consisting of excitable elements.

References

[1] I. Ratas and K. Pyragas, Phys Rev E 100, 052211 (2019);

[2] I.V. Tyulkina, D. S. Goldobin, L. S. Klimenko, and A. Pikovsky, Phys. Rev. Lett. 120, 264101 (2018).