Enhancement and suppression of pulse propagation in a discrete Fitzhugh-Nagumo model subjected to a high-frequency stimulation

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Motivation

Excitable systems have only one stable fixed point, but perturbations above a certain threshold induce large excursions in phase space, which take the form of spikes of fixed shape. Diffusively coupled excitable systems may produce propagation of this excitation, i.e. pulses traveling through neurons. Our aim is to study how an external high-frequency perturbation effects such systems and try to understand the effects of deep-brain stimulation to neurons.

Model

The myelinated axon is modeled by a one-dimensional chain of diffusively coupled excitable elements described by the FitzHugh-Nagumo (FN) equations:

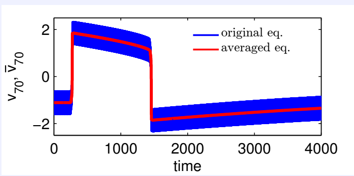
$$\begin{aligned}\dot{v}_n &= f(v_n) - w_n + D(v_{n+1} - 2v_n + v_{n-1}) + a\cos(\omega t),\\ \dot{w}_n &= \varepsilon(v_n + \beta - \gamma w_n).\end{aligned}$$

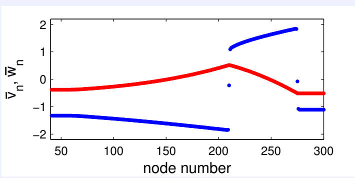
Here the function $f(v_n) = v_n - \frac{1}{3}v_n^3$, v_n - the membrane potencial, w_n- the recovery variable, a, ω - the stimulation amplitude and frequency.

Tools

1. Two-scale method [1]: The period of the stimulation is much lower than the lowest characteristic time-scale of the free system. This method allows to separate the slow and fast motions, therefore autonomous equations for slow motion can be derived.

2. Asymptotic pulse construction [2]: Derived autonomous equations enabled us to use known methods for pulse parameters estimation at the limit of slow recovery ($\varepsilon \to 0$). In this case pulse can be subdiveded in four parts, which can be described by simplified averaged equations.



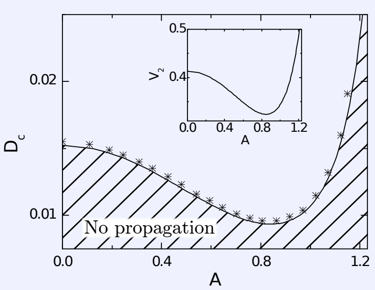


Results

The deviations from averaged equations resting point are governed by

$$\begin{aligned}\delta\dot{v}_n &= F(\delta v_n) - \delta w_n + D(\delta v_{n+1} - 2\delta v_n + \delta v_{n-1}),\\ \delta\dot{w}_n &= \varepsilon(\delta v_n - \gamma\delta w_n),\end{aligned}$$

Where $F(\delta v_n) = -\delta v_n(\delta v_n - V_1(A))(\delta v_n - V_2(A))/3$, $A = a/\omega$ - stimulation intensity and $V_1 > V_2$.

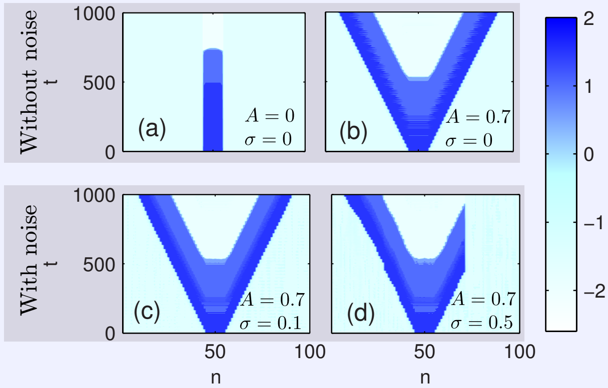


The front of the pulse cannot propagate in the system if its diffusion coefficient is smaller than some critical value. The critical diffusion coefficient can be estimated by usig regular perturbation theory:

$$D_c \approx V_2^2(A)/12$$

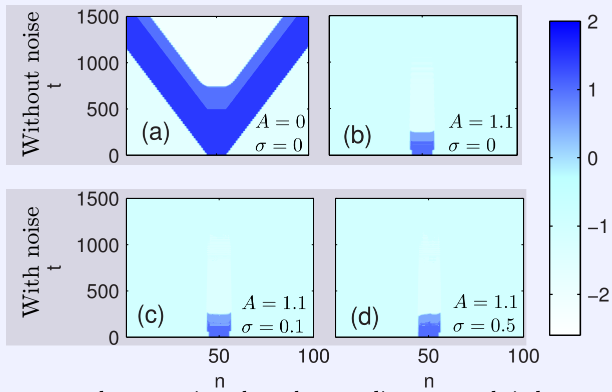
Here the solid curve and asterisks show respectively the analitical and numerical estimations of the critical diffusion coefficient.

Pulse enhancement



Spatio-temporal evolution of the membrane potential obtained from initial equations. The background of homogeneous high-frequency oscillations is excluded i.e. the color encodes the value $v_n - A\sin(\omega t)$. The fixed parameters are: $\omega = 10$, $\varepsilon = 0.0008$ and D=0.015. The Gaussian noise was added to the recovery variable.

Pulse suppression



The same graph as previous but the coupling strength is larger D=0.02.

Conclusions

- As in continuous case [3], the high frequency stimulation (HFS) suppresses the pulse propagation when stimulation intensity exceeds a critical value.
- HFS may enhance the pulse propagation when without stimulation the system demonstrates propagation failure. This effect is determined by nonmonotonical dependence of the system's excitability parameter on stimulation intensity.
- The effect of suppression of pulse propagation is less sensitive to the noise than the effect of enhancement of propagation.

Acknowledgments

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References

[1] J. K. Kevorkian and J. D. Cole, Multiple scale and singular perturbation methods (Springer-Verlag, 1996)

[2] A. Caprio and L. L. Bonilla, SIAM Journal on Applied Mathematics 63, 619 (2002)

[3] I. Ratas and K. Pyragas, Nonlinear Dynamics 67, 2899 (2012)